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I.—THE MAGNETISM OF ANNEALED CARBON STEELS.

By Prof. S. W. J. Smith, *F.R.S.*, A. A. Dee, *B.Sc.*, *D.I.C.*, and W. V. Mayneord, *M.Sc.*, University of Birmingham.

Read May 23, 1924.

Abstract.

This Paper is a sequel to one by S. W. J. Smith and J. Guild (Proc. Phys. Soc., 1912, Vol. 24, p. 342) on the "Self-Demagnetisation of Annealed Steel Rods," in which it was shown that the residual moment of a magnetised rod of annealed steel (of particular dimensions) does not diminish continuously towards zero as the temperature is raised, but changes sign at about 180°, reaches a maximum negative value at about 220°C., and subsequently slowly falls until the temperature at which steel ceases to be ferromagnetic is attained.

Experimental evidence is adduced to show that the reversal is primarily due to the laminated structure of the iron-iron carbide "eutectoid" which the steel contains.

It is shown that the reversal disappears when this structure is destroyed, and an explanation of the fact that the effect is most pronounced in a steel containing the eutectoid percentage of carbon is given.

Measurements have also been made to find how the intensity of the residual moment at 220°C., depends upon (1) the dimension-ratio of the bar, (2) the intensity of the field by which it is magnetised. The results are described and an interpretation is given.

It is suggested that measurements of the kind described could be employed to obtain rapidly and in a simple manner useful information concerning particular samples of steel.

I. Introductory.

IN the years just preceding 1914, one of us, assisted by others, examined various properties of "steel" arising from the fact that it is a mixture of two or more substances each with its own magnetic characteristics.

It was pointed out in connexion with the results obtained that various problems of interest remained to be investigated. To these until recently it has not been possible to return. On the present occasion we desire to describe a few of the further experiments that have been made. These may be regarded as a continuation and elucidation of those described in the Paper on the "self-demagnetisation of annealed steel rods" read before the Physical Society in 1912.

Among other things, we hope to show that a criticism of these experiments which has been offered* rests upon a misunderstanding of their significance.

Although our interpretation of these particular experiments is practically the same now as it was then, it is possible that, in the desire to be brief, we did not express it as clearly as we might have done. We hope, however, that the new experiments about to be described, together with our explanation of them, will leave no room for doubt concerning the cause of the phenomena we described.

II. The Constitution of Annealed Carbon Steels.

Annealed carbon steels are magnetic mixtures of which the constitution and micro-structure are now well known.

Disregarding the "impurities," chiefly manganese and silicon, in those we have used, such steels contain only iron and its carbide Fe_3C. The amount of the latter contained in a particular steel is directly proportional to the percentage of

* N. H. Williams, Phys. Rev., III, 115 (1914).

carbon which it contains. If the steel contains c per cent. of carbon, the percentage of carbide present is approximately 15 c.

Instead of being uniformly distributed throughout the mixture, the carbide is intimately associated with a definite proportion of the iron. It is distributed with respect to this associated iron in alternating, roughly parallel layers. This complex of the two substances is the eutectoid mixture "pearlite" containing as a whole e per cent. of carbon, where e is approximately 0·9.

If the steel contains exactly the eutectoid percentage of carbon, it consists entirely of grains of the complex.

If c is less than e, i.e., if the steel is "hypo-eutectoid," it contains grains of the complex together with grains of "free" iron. The proportion in which these are present is such that the eutectoid grains constitute the fraction c/e of the whole mass.

If c is greater than e, i.e., if the steel is "hyper-eutectoid," it contains grains of the complex together with grains of "free" carbide, the fraction of the whole mass which consists of eutectoid grains being $(100-15c)/(100-15e)$.

The percentage of carbon contained in the steels examined by Mr. Guild and one of us ranged from about 0·15 in that containing least to about 1·5 in that containing most. In the former about one-sixth was eutectoid, the remainder being iron. In the latter about nine-tenths was eutectoid, the remainder being carbide. In the first, grains of pearlite occur in patches separated by grains of iron. The second consists of grains of pearlite together with grains of excess carbide, many of which occur as a network surrounding the pearlite grains.

III. Experiments Suggested by the Fact that Steel is a Magnetic Mixture.

The carbide, like iron, is ferro-magnetic; * but, compared with the latter, it is magnetically hard. The field required to produce saturation in it (as it occurs in steel) is much greater than in iron and the retentivity is correspondingly greater.†

It seemed obvious that a careful study of the magnetic properties of a mixture of such substances would lead to interesting results.

One of the problems which suggested themselves was this: What will be the distribution of magnetism in a rod of such material, magnetised in the ordinary way? How much will the iron and the carbide, respectively, contribute to the moment of the rod?

It was thought that part at least of the answer to such questions might be found by making use of the fact that the carbide ceases to be ferro-magnetic‡ when heated above 215°C. It was proposed to measure the moment of the rod at room temperature and then to heat it continuously, observing how the moment varied. It seemed clear that some change in the slope of the magnetism-temperature curve would occur near 215°C. Below this temperature both constituents would be contributing to the observed moment; above it the iron would be acting alone.

* cf. Smith, Proc. Phys. Soc., 25, 77 (1912).

† cf. Smith and Guild, Proc. Phys. Soc., 24, 345 (1912).

‡ Smith, White and Barker, Proc. Phys. Soc., 24, 64 (1911); and Smith, Proc. Phys. Soc., 25, 77 (1912).

The results of the preliminary experiments arising out of these considerations are shown in Fig. 1 of the Paper by Smith and Guild (l.c., p. 344). Six rods containing different percentages of carbon, ranging from 0·15 to 1·53, were examined. They were all of the same dimensions, namely, 7 cm. long and 0·55 cm. in diameter. Each had an axial hole, of diameter 0·3 cm., drilled through it from end to end for convenience of temperature measurement. The impurities, chiefly manganese and silicon, were present to about the same amount in all.

In each case the moment fell at first, as expected, when the temperature rose; but, except in the steel containing the smallest percentage of carbon, the variation of the moment at the higher temperatures was quite different from what seemed likely to happen before the experiments were made. In every other steel the moment fell continuously until, in the neighbourhood of 180°C., it became zero. It then changed sign, ultimately reaching a maximum negative value near 220°C. From this it rose again towards zero, reaching that value when the temperature* at which steel loses its magnetism was attained.

IV. The Residual Magnetism of Annealed Steel Rods.

The fact that the negative magnetism reached its maximum value at the point where the magnetism of the carbide disappeared made it obvious that it was carried by the iron. It remained to discover how this negative magnetism was acquired.

It was clearly not due to a reversal of the type discovered by Waltenhofen, for it was obtained when the conditions which this reversal requires were absent. This was stated quite explicitly in the Paper in which the experiments were described; but, according to Williams, the reversal only occurred in the rod he used when the Waltenhofen conditions were present, i.e., when the current used to magnetise his rod oscillated at break.

The rod he used had a very different dimension-ratio from ours and, although it contained 0·85 per cent. of carbon, nothing is said as to the conditions under which it was annealed. Williams attributes the reversal of moment beyond 180°C. to what happens in the outer layers of the rod and suggests that these will keep their magnetism longer than the interior when the temperature is raised because it is "almost inevitable that steel with such a high percentage of carbon should be slightly harder in its outside layers than at the interior of the specimen." Continuing, he says: "We should then expect that with rise of temperature the magnetism of the outer layers would persist after that of the softer interior had disappeared and hence that the resultant magnetic moment would be reversed at the high temperature."

Disregarding the fact that the reversal was obtained in steels containing percentages of carbon much lower than 0·85, it may be pointed out that, unless precautions are taken to avoid decarburisation during annealing, the outer layers may easily be magnetically softer than the interior. Apart from this, the proposed interpretation leaves unexplained the fact that the reversal occurs at a temperature evidently closely related to the transition temperature of the carbide.

Soon after the results obtained by Williams were published, one of us had a rod constructed, from steel of unknown composition, of which the dimensions were

* Smith and Guild, Phil. Trans. A, 220, 177 (1914).

nearly the same as those of the rod he used. This rod, which was 7 cm. long, had a cylinder hole just exceeding 0·55 cm. in diameter drilled through it axially. Into this could be fitted closely a rod of the same steel whose diameter was 0·55 cm. and length 7 cm. This composite rod showed no reversal of magnetism when heated, whether magnetised by the method originally used or by slowly withdrawing it from a solenoid according to the method adopted by Williams. Neither did the larger part of the composite rod when separately magnetised. The composite rod was next remagnetised as before and the inner rod was now withdrawn. On being tested it gave a reversal of moment of the kind originally described. It is obvious that neither outer skin nor eddy currents could be the cause of this result.

V. The Cause of the Change in Sign of the Residual Moment.

We shall now attempt to show, more explicitly than in the earlier Paper, how, in our opinion, the reversal is produced.

The primary cause of the effect is the lamination of the eutectoid, i.e., the existence within the steel of alternating thin plates of two different substances, one magnetically hard and the other magnetically soft.

The fact that the negative magnetism is most pronounced in the steel containing nearly 0·9 per cent. of carbon is enough to suggest this. It is the only steel which is composed entirely of laminæ. But, as we shall see, much more direct proofs than this can be found. Before turning to these, a few remarks may be made concerning the effects which lamination might be expected to cause.

To take the simplest case, imagine two rods of the same size, one magnetically hard and the other magnetically soft, placed side by side to form a composite bar. Let this bar be magnetised by placing it in a field parallel to its length. Upon withdrawal of the field, each half will be subject to the demagnetising influence of its own residual magnetism. The extent to which this is effective will depend upon the ratio of the length of each rod to its breadth. The shorter the rod in comparison with its breadth the more effective will the demagnetising field be; but no rod, however short, can reduce its own moment to zero. Still less can it reverse the sign of that moment. But, besides acting upon itself, each rod acts upon the other. It is the rod of the relatively soft material that fares worst in this encounter. Hence it is that the polarity of the softer material may be reversed while that of the harder retains its original sign.

The conditions under which reversal of this type occurs can easily be studied by means of a model consisting of rods of iron and steel respectively. Experiments of this kind made for a particular purpose are described later in § X.

It can be seen that a composite bar consisting of alternate laminæ of hard and soft materials will possess magnetic properties similar in character to those of the simple model described above. Models to represent such cases are not difficult to construct and to use. For example, to show, as could be predicted, that the reversal produced by a given field in a rod of given size is more pronounced when the laminations are fine than when they are coarse.

The magnetic conditions within a rod of steel can never be as simple as in models of the kind just described since it probably never happens that the orientation of the iron and the carbide is the same in all the grains of eutectoid which the steel contains. In a fraction only of these will the laminæ lie in planes containing the direction of the magnetising field. In all others, the effects produced will be

indirect, the extreme case being that of the grains whose laminations are perpendicular to the direction of the field.

We can, however, form a rough picture of the magnetic structure of any rod by regarding it as equivalent to a series of undulating laminated surfaces of which some are interrupted at different places in their course from one end of the rod to the other.

It would not be impossible to construct and examine a model with a structure of this character; but for our present purpose it is unnecessary to do so, because it is easy by thermal treatment to convert a steel in which the lamination is complete into one in which there is none. If in the latter case reversal does not occur, we obtain a direct proof that lamination is its cause.

VI. Experimental Proof that Reversal does not occur when Laminations are Absent.

We can begin by taking, for example, a eutectoid steel in which, by moderately slow cooling from about 750°C., the laminated structure, produced at about 700°C., is fully developed and observing the subsequent variation of its residual magnetism with temperature rise. The same specimen can now be heated to about 750°C. and cooled suddenly by quenching in water. By this means the lamination is suppressed. After this treatment the specimen is heated to a temperature not exceeding 700°C., maintained at this temperature for a few hours and then allowed to cool to room temperature. It is known from microscopic examination that the steel now consists of granules of carbide in a matrix of iron. If our interpretation of the phenomena be correct, the steel in this condition should show no reversal of residual magnetism when, after magnetisation at room temperature, it is heated to 200°C.

Experiments of this kind were performed upon a steel containing about 0·8 per cent. of carbon, with the results shown in Fig. 1.

The rod was of the same dimensions as those used in the earlier experiments. The abscissæ represent temperatures and the ordinates residual magnetism (in arbitrary units) as before.

The lowest curve shows the behaviour of the steel after slow cooling from 780°C. Direct microscopic examination, after the usual polishing and etching, showed that the steel consisted almost entirely of lamellar pearlite. The reversal of sign of the residual moment occurs exactly as in the earlier experiments.

The other curves represent the data obtained when, after quenching, the same rod had been heated for some time to temperatures of 400°C., 500°C., 600°C. and 680°C., respectively, having been allowed to cool to room temperature between successive heatings.

In none of them, as will be seen, does reversal occur.

After the experiments the specimen was again examined microscopically and seen to contain no laminations. It possessed the structure already described as characteristic of steel which has undergone thermal treatment of the kind to which it was subjected.

It is, of course, easy to see why reversal should not occur in steel of this structure. Its properties are now, in fact, very like those one of us half expected to find, as stated in § III., before any experiments upon this subject were made.

VII. Practical Applications.

When the results shown in Fig. 1 are regarded from the practical standpoint, it will be seen how easy it is to determine by this magnetic method of examination whether a particular steel is in the laminated or in the granular condition. It is of interest in this connexion to note that the character of the laminæ in a particular steel can be altered considerably by thermal treatment less drastic than quenching, with subsequent tempering.

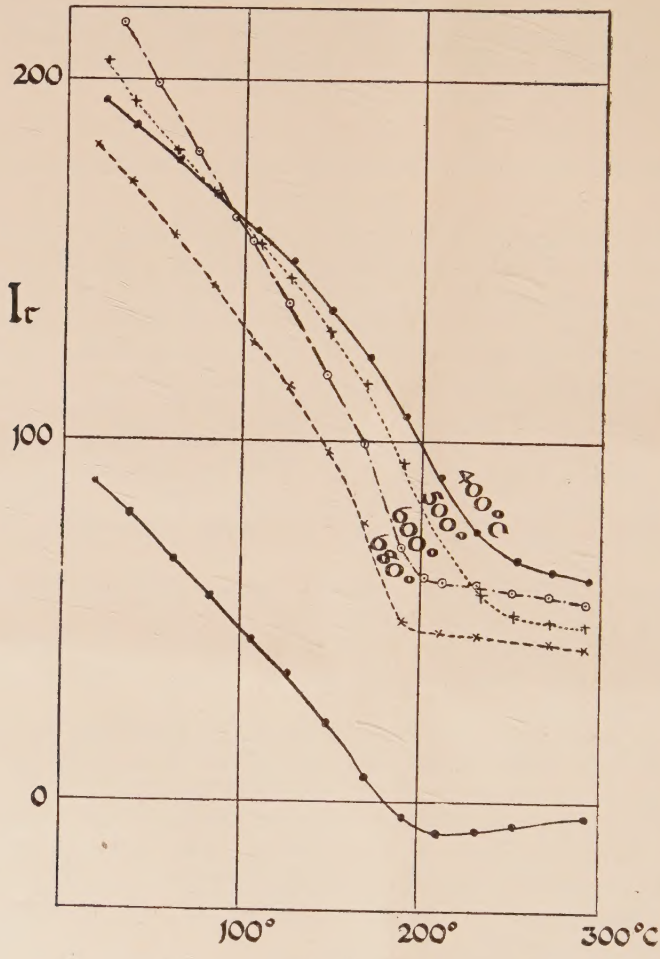


Fig. 1.

For instance, when steel after heating to 780°C. is taken out of the furnace and cooled in air instead of in the furnace, it is found by microscopic examination that the laminæ are of finer texture than those which result from the slower rate of cooling. On the other hand, if the steel during cooling from 780°C. is kept for a long time at a temperature just below that at which the laminations are produced, it is found that these laminations gradually break up under the influence of surface

forces which give rise to structural instability. In consequence, steel treated in this way is partly laminar and partly granular when allowed to cool, and therefore intermediate in structure between an ordinary annealed steel and one tempered after quenching.

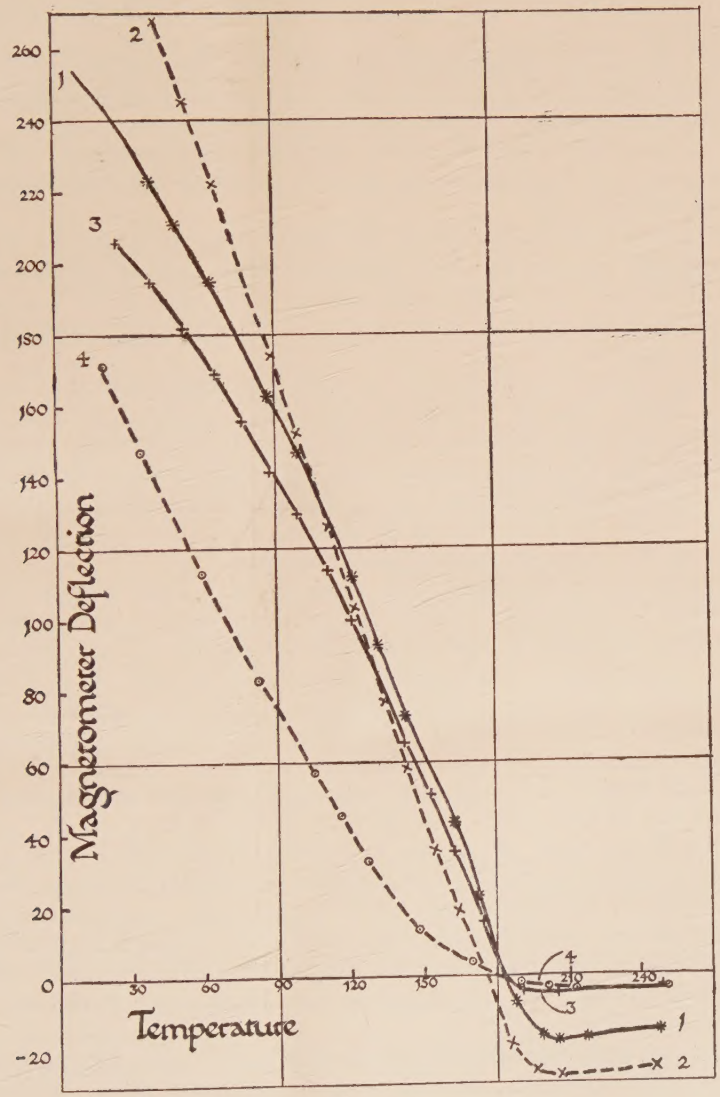


FIG. 2.

Experiments were made to find how such structural differences influence the temperature variation of residual magnetism.

Some of the results are shown in Fig. 2, which relates to a cylindrical rod of

steel containing 0·9 per cent. of carbon, together with 0·29 per cent. of manganese, and 0·27 per cent. of silicon. Its length was 7 cm., and its radius 0·72 cm. A hole of 3 mm. diameter, to take the thermo-couple, was drilled through it axially.

It was magnetised in a solenoidal field of about 800 c.g.s. units, from which it was withdrawn in each experiment while the current was still flowing. The ordinates represent the residual moment of the rod in arbitrary units, the same in each experiment.

Curve 1 shows the behaviour of the steel after heating to 780°C., keeping at this temperature for about twenty minutes and then allowing to cool in the furnace, about thirty minutes elapsing before the temperature reached 400°C.

Curve 2 shows the behaviour of the same steel after heating to 780°C., keeping at this temperature for about twenty minutes as before and then taking out of the furnace and cooling in air.

Curve 3 shows the behaviour of the steel when, after heating to 780°C., the temperature was lowered and held between 650°C. and 670°C. for eight hours. The great reduction in the reversed magnetic moment (above 180°C.), corresponding with the partial disintegration of the laminæ, is apparent when Curve 3 is compared with Curves 1 and 2.

It is known that " cold working " of steel containing laminated pearlite, before heating to 680°C., greatly accelerates the rate of disintegration of the pearlite. Corresponding with this, it is found that the reversal disappears in such steels when, after cold-working, they are re-heated and kept for a comparatively short time at 680°C. We omit further reference to experiments of this kind for the present. It is sufficient to say that the effects of thermal treatment upon internal structure can be seen at once when the magnetism is measured.

Curve 4 shows the behaviour of the steel of Fig. 2, after heating to 950°C. and keeping at this temperature for three hours with subsequent cooling in the furnace. This treatment is known to lead to a characteristically coarse structure in which " minimum interlocking of the iron and the carbide " occurs. It is seen in this case that the reversal of residual magnetism has again almost disappeared.

This Curve exhibits another interesting feature when compared with that obtained after annealing at 780°C. The angle which the tangent to the curve makes with the horizontal axis diminishes continuously as the temperature rises. The curve shares this property with that given by Williams, for a rod of " annealed steel containing 0·85 per cent. of carbon " of similar dimensions. This suggests that the annealing process applied by him, of which no details are given, was not of the kind used by us in the experiments which he criticises. Apart from this, it is important to note that even when the treatment we described is applied to a rod, like that of Fig. 2, of dimensions similar to that used by Williams, the reversal is much less pronounced than when the dimensions are those given in the Paper to which he refers.

It is curious that Williams did not attempt to reproduce our conditions exactly, when examining our results, instead of endeavouring to show that oscillations at break of the magnetising current would produce or accentuate the reversal. This point was never in question. What we stated was that reversal occurred, under specified conditions, when such oscillations were avoided in the way we described. It also occurs when the particular method of avoiding oscillations described by him is used.

VIII. The Dependence of the Reversed Moment upon the Percentage of Carbon Present.

Before discussing how the dimensions of the rod affect the results, we may indicate briefly how the extent of the reversal may be expected to depend upon the percentage of carbon which the steel contains.

The amount of "free" iron, not associated with the carbide as eutectoid, increases continuously as the percentage of carbon falls below 0·9. Since the carbide of the eutectoid cannot in general produce reversal in this iron, it can be seen that the negative moment at 215°C., representing that of the reversed iron, will become an increasingly smaller fraction of the original residual moment at 15°C. as the percentage of carbon present is reduced.

Similarly when the percentage of carbon increases above 0·9, increasing amounts of "free" carbide appear. Some at least of this will be disposed with respect to the eutectoid grains as an enveloping net work, i.e., in such a way as to lessen the reversing effect of their carbide laminæ upon the iron laminæ which they contain.

It would also appear that the laminæ of hyper-eutectoid steel are frequently coarser in texture than those of hypo-eutectoid steel. This might also diminish the effectiveness of the carbide laminæ in producing reversal of the associated iron.

Further investigations of such points might be made experimentally, but from the experiments already described and the general considerations just given, it is probable that in a series of steels, similar in character and in previous thermal treatment, the maximum reversal will occur in that containing the eutectoid percentage of carbon.

It is, of course, easy to see why, in a completely granulated steel, reversal will in general be absent. Sometimes, however, there may be a regular linear distribution of granules which can produce reversal. It may also happen, in hyper-eutectoid steel, that, although pre-existing pearlite has been granulated, the excess-carbide has escaped complete granulation. In such steel, it is possible that reversal may occur. We have made a beginning upon experiments concerned with such questions; but need not refer to them further at present.

IX. The Dependence of the Reversed Moment upon the Dimension-ratio of the Rod.

We turn now to the consideration of the question: How should the intensity of the residual negative polarity shown at 215°C. depend, in a given steel, upon the dimension-ratio of the rod employed?

We can see, in a general way, what answer to expect by considering again the simple model of hard and soft rods placed side by side. When the length of the composite rod is great compared with its diameter, i.e., when the dimension-ratio l/d is large, the demagnetising effect of each half upon itself as well as upon the other will be relatively small. Hence we should not expect reversal in either.

Conversely, when the ratio l/d is very small, the demagnetising effect of each half upon itself will be so great that even in the harder half the resultant magnetic intensity will be only a small fraction of that existing before the magnetising field is removed. Consequently the negative intensity which this half can induce in the other must also be small, since the polarity of the rod as a whole cannot change sign. What happens in these and in intermediate cases can be followed in greater

detail by considering the I H curve for each material in conjunction with the demagnetisation coefficients, experimentally known, for each dimension-ratio.

Applying such considerations to steel, regarded as a laminated substance, it can be seen that the residual negative intensity at 215°C. will pass through a maximum value as l/d is decreased continuously, i.e., there will be a particular dimension-ratio for which the phenomenon of reversal is most pronounced.

In Fig. 3 are collected the results of experiments made to test this inference.

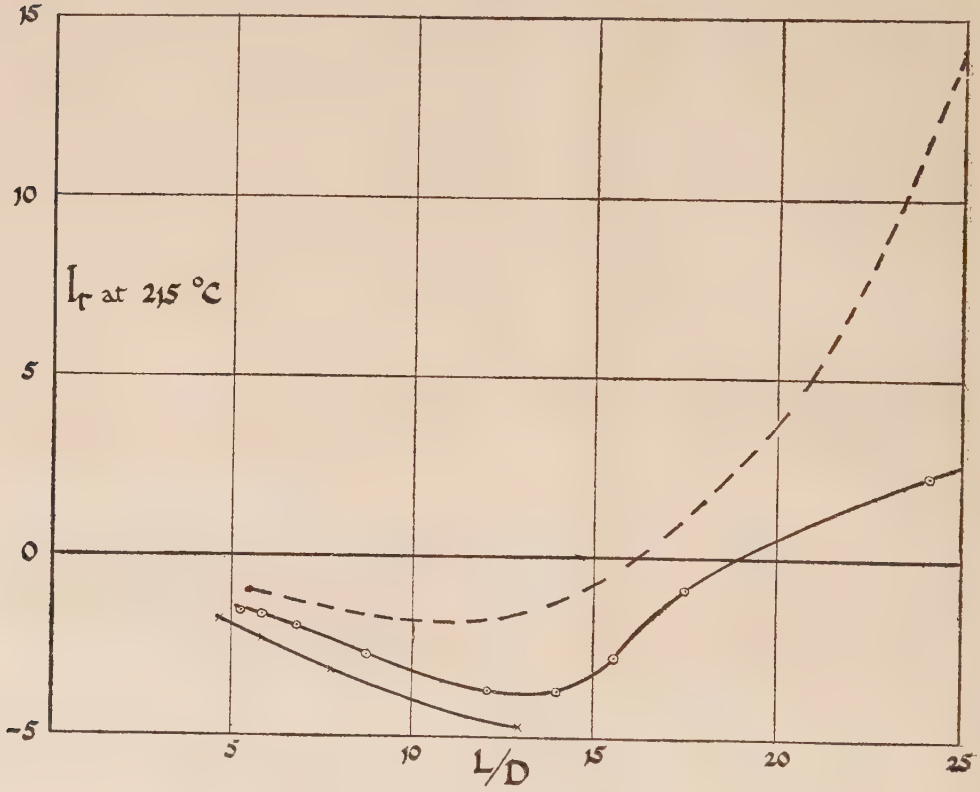


FIG. 3.

The dotted curve represents a few preliminary results obtained by using a rod of tool-steel containing about 1 per cent. of carbon, about half a centimetre in diameter and originally about 20 cm. long. At its original length, the residual intensity at 215°C. was positive and about thirty times as great as the maximum negative intensity shown in the Figure. This residual intensity fell rapidly as the rod was shortened; but the measurements made were so widely spaced that they serve only to indicate generally what happens. It is seen, however, that somewhere between $l/d=20$ and $l/d=15$, the residual magnetism changes sign and that the negative value is less when $l/d=5$ than when it is 13.

In this case the values of the residual magnetism are given in arbitrary units chosen for convenience of representation on the same diagram as the other curves. In these, the intensity of magnetisation is expressed in c.g.s. units.

The lowest curve represents experiments made upon a rod of which the length was kept constant at 7 cm., while the dimension-ratio was increased from one measurement to the next by reducing the diameter on the lathe.

This steel had the same composition as that used in the experiments of Fig. 2 and was in the condition following slow cooling from 780°C. as in Curve 1 of that Figure. The magnetisation at room temperature was carried out in the same way as in the experiments of Fig. 1. The initial diameter of the rod was 1·5 cm. The final diameter was 0·54 cm. The initial and final dimension-ratios were thus approximately 4·7 and 13.

It will be seen that the negative intensity increases at a gradually decreasing rate as the dimension-ratio is increased. The original ratio approximates to that used by Williams, while the final one approximates to that used by Smith and Guild. The rod had the usual axial hole of 0·3 cm. diameter.

The middle curve shows a more complete series of observations obtained in another way.

In this case the experiments were made upon a rod of adjustable diameter consisting of a solid core and a series of concentric tubes fitting closely upon one another, made from the same steel. This contained 0·77 per cent. of carbon, the core and tubes being annealed beforehand by cooling in the furnace from 780°C. The length and diameter of the core were 7 cm. and 0·29 cm. respectively. The tubes were of the same length as the core, and when all were in position, the arrangement was equivalent to a rod 7 cm. long and 1·34 cm. in diameter.

It is not to be expected that this method of procedure will give exactly the same results as would be got from a rod of the same material turned down in the lathe between successive measurements of the residual intensity. It is, however, of interest from the fact that the conditions of annealing are different and can be altered at will, and that observations with a given dimension-ratio can be repeated at any time during the course of the experiments. In the particular set of measurements represented, the existence of a maximum negative intensity is clearly shown. It occurs when the dimension-ratio is about 13. The negative polarity disappears when l/d exceeds 19.

X. The Dependence of the Reversed Moment in a Laminated Rod upon the Magnetising Field.

One other question remains: How, for a given rod of steel, does the sign and intensity of the residual magnetism at 215°C. depend upon the strength of the field used to magnetise it at 15°C. ?

The answer can, of course, be found by direct experiment; but before giving the results obtained it is convenient to examine the behaviour of a model of the kind already described, consisting of rods of hard and soft materials placed side by side. The results obtained using steel can afterwards be compared with those obtained in this simplest possible case of lamination. The extent to which the views expressed in preceding sections are confirmed by experiments of this type will then be clear.

The results shown in Fig. 4 were obtained by the use of a composite cylindrical rod, consisting of half-cylinders of iron and steel respectively, each 7 cm. long and of 0·3 cm. radius, in contact with each other along their plane rectangular surfaces.

For each experiment this composite rod, after demagnetisation of each half,

was magnetised by means of a solenoid through which a measured current was passing. After withdrawal from the solenoidal field, the residual moment of the bar was measured in arbitrary units. The brass clip which held the two halves together was now removed and they were carefully separated, being kept parallel to one another while their centres moved apart along a line perpendicular to the direction of magnetisation. The residual moment of each half was now measured in the same way as that of the whole rod.

The results, for different values of the solenoidal field in c.g.s. units are shown in the Figure.

It would take too long to deal with the significance of every detail of these results. We shall confine ourselves to their general interpretation, which is all that is required for present purposes.

In the weakest fields as will be seen, the residual moment of the iron half is

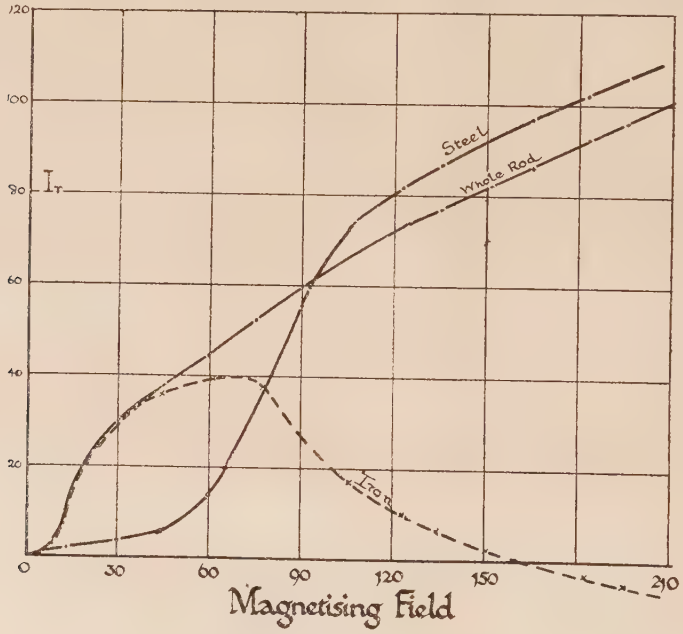


FIG. 4.

practically the same as that of the whole rod, while that of the steel half is relatively very small. On account of its much greater magnetic hardness, the response of the steel to such magnetising fields is very small compared with that of the iron. When the field is withdrawn, the steel's own demagnetising field aided by that of the iron reduces its moment practically to zero. The demagnetising field on the iron is thus practically the same in the composite bar as it is when the steel is removed. Consequently its moment will remain practically unchanged upon that removal and will be same as that previously shown by the whole rod. That of the steel, however, will rise a little in consequence of the removal of the reversing field previously exerted by the iron. Accordingly, as will be seen, the sum of the moments shown

by the two halves will be, from the outset, greater than that shown by the rod as a whole.

Although the residual moment of the softer half rises rapidly, at first, as the magnetising field is increased, the rate of rise soon slackens and simultaneously that of the steel half begins to increase. The " second stage " in the magnetisation curve for the iron is nearing completion when that for the steel begins.

In a field slightly less than 80 c.g.s. units, the residual moment of the steel has become equal to that of the iron, the sum of the two being now much greater than

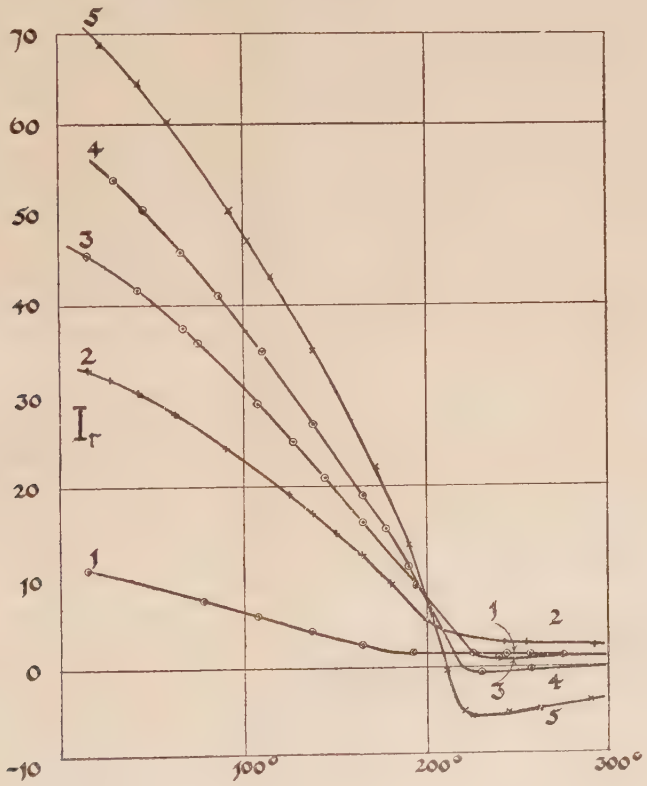


FIG. 5.

that of the rod as a whole. The moment of each half has evidently risen considerably when the demagnetising field due to the other has been removed by separating them.

From this point the residual moment of the iron falls rapidly, while that of the steel continues to rise. After magnetisation in a field of about 90 units, the residual moment of the steel is as great as that of the whole rod, while that of the iron is less than half this amount, being at the same time, no doubt, considerably greater than it was before separation from the steel. After magnetisation in a field of about 160 units, the residual moment of the iron is zero, although, no doubt, it was negative before the demagnetising field due to the steel was removed; just as the positive moment of the steel is presumably less positive now than it was before the iron was taken away.

Finally, in still larger fields, the residual moment of the iron is negative. The

reversal produced by the steel is now so great that it persists when the latter is removed.

XI. Experimental Results for Annealed Eutectoid Steel.

The experiments, of which the results are collected in Fig. 5, were made upon a rod 7 cm. long and 0·55 cm. diameter, containing 0·85 per cent. of carbon. The ordinates represent residual moments in arbitrary units, the abscissæ being temperatures in degrees Centigrade.

The curves show how the residual moment varied while the temperature was raised to 300°C., after magnetisation at about 15°C. in fields of different intensities. Curve 1 represents the behaviour of the steel after magnetisation in the weakest field that could be employed conveniently. Curve 5 was obtained after magnetisation in a field strong enough to produce approximate saturation. Intermediate fields gave the results shown in Curves 2, 3 and 4.

It will be seen that the magnetism shown by the iron, above the temperature at which the carbide ceases to be ferro-magnetic, is positive when weak magnetising fields are used and rises at first when the field intensity is increased (cf. 1 and 2). Subsequently it falls again (cf. 2 and 3); ultimately becoming negative to an extent which increases as the field strength is raised (cf. 4 and 5).

Except for the fact that the iron which it contains is heated, raising the temperature of the steel to 220°C. separates the iron from the carbide in practically the same way as it was separated from the steel in the composite rod of § X. Accordingly, if annealed steel owes its magnetic properties to alternating laminæ of iron and its carbide, relatively soft and hard at 15°C., the interpretation of its behaviour in the experiments of Fig. 5 presents no difficulty.

XII.

Further consequences of the existence of this condition in steel have suggested themselves and have been examined. Of these perhaps the most interesting occur when the steel is magnetised at temperatures above the critical temperature of the carbide and then allowed to cool. These experiments were begun by one of us and Mr. Guild.* We hope to describe them upon some future occasion.

DISCUSSION.

Mr. J. Guild said that it came as a pleasant surprise to him to find that the Paper dealt with an extension of the work carried out some years ago by Prof. Smith and himself—and he was very glad that their original explanation of the facts had been vindicated so clearly. This explanation when first put forward had been rather contumeliously criticised, particularly by Williams, who proposed an alternative theory. A number of phenomena could be quoted in support of the original view, but instead of detailing these he would refer to some further work which had not been published. Williams had tried to repeat some of the experiments with a large thick rod, and had found that the polarity-reversal effect occurred if the magnetising field was broken suddenly but not if the specimen was gradually withdrawn from the field. He based a theory on this fact and wished to explain the original experiments by means of it, but as a matter of fact the conditions assumed by him did not obtain in the experiments in question. The speaker tried to repeat Williams' results with rods of various sizes, and found that their character was reversed when small rods were used. He had been able to trace the effect of the dimensions of the specimen on the results obtained, but had been obliged by the pressure of other work to abandon the investigation. The fresh light on the matter contained in the Paper now before the meeting should make it possible to explain this effect.

The President thanked the authors for an interesting and conclusive Paper. He expressed the great pleasure felt by the Fellows present at having Prof. Smith once more amongst them.

* Smith and Guild, l.c., pp. 346, 7.

II.—UNDERBLOWN PIPES.

By D. Gunnaiya, *M.A.*, and G. Subrahmaniam, *M.A.*, H.H. The Maharajah's College, Vizianagaram.

Received June 3, 1924.

(*Communicated by* Lord Rayleigh.)

Abstract.

The phenomena of underblown pipes has been studied in the case of four wooden pipes ranging over an octave. It has been found that it presents many features of remarkable interest. The existing theories regarding the phenomena have been discussed, and a new theory is proposed which attempts at once to explain both these phenomena.

Introduction.

THE behaviour of flute organ pipes with different blowing pressures presents peculiarities which have so far been mathematically unaccounted for. It is a matter of common knowledge that an organ pipe gives out its fundamental note with a particular blowing pressure. This pressure to elicit the fundamental is, however, different for different pipes, and even with the same pipe it largely depends on the details of its make and the distance of the slit from the lip. Though the phenomena of overblown pipes have been known for a long time, no definite advances in theory appear to have been successful in explaining them. In recent years some consideration has been given to this subject by Arthur C. Lunn,* V. Lough,† and Bhargava and Ghosh.‡

A. C. Lunn indicated in his Paper a short theory of the phenomena of overblowing, which for want of necessary experimental data could not be verified satisfactorily. The rough values which he himself has given are found to be at variance with the theory put forward. The present Paper is the result of an attempt on the part of the authors to obtain sufficient data whereby to test the validity of his theory. In the course of their experiments they have found that, even before the pressure rises sufficiently to elicit the fundamental, the organ pipe gives out notes, no less interesting than those of the overblown. Hence it is proposed to examine these phenomena of "underblowing" in greater detail and to put forward a theory which shall embrace the phenomena of overblowing also. The last-mentioned effect was for the first time recognised by the late Lord Rayleigh,§ who with a metal open pipe of frequency 255, and using the method of Blaikley,‖ arrived at the variation of pitch under a gradually decreasing blowing pressure.

* Phys. Rev., May (1920).
† Phil. Mag., Jan. (1922).
‡ Phys. Rev., Nov. (1922).
§ Phil. Mag., 13 (1882).
‖ Phil. Mag. (1879).

Experimental.

The experiments were conducted with four wooden open pipes of frequencies 256, 320, 384 and 512. The pressures, obtained by an acoustic bellows with a large wind-chest, were gradually increased, and could be maintained constant at any value. At any instant the pressure was measured by a water manometer, using a cathetometer microscope. The pitch of the note at any stage was found by tuning a monochord to unison by an expert musical ear. The values are tabulated below :—

Table of Values.—*Pressures are in cm. of water.*

Pitch of the pipe.	Underblown tone corresponding to fundamental.				Underblown tone corresponding to octave.				Natural fundamental commences at		Pressure for frequency equal to the actual face value.
	Commences at		Ceases at		Commences at		Ceases at				
	Pressure	Frequency	Pressure	Frequency	Pressure	Frequency	Pressure	Frequency	Pressure	Frequency	
256	0·10	247·9	0·65	271·0	0·65	529·3	1·4	550·7	1·4	245·8	2·0
320	0·20	283·0	0·80	341·7	0·80	631·9	2·4	...	2·4	297·3	3·2
384	0·30	281·4	1·20	402·2	*Elicited with difficulty.*		*Uncertain.*		3·2	312·4	4·2
512	0·90	507·0	2·2	539·5	*No octave*		*counterpart.*		6·2	482·7	9·2

Conclusions.

From the above results we are led to conclude that the behaviour described by Lord Rayleigh is characteristic of pipes with frequencies lower than a limiting frequency. The behaviour of the 256 pipe is similar to that described by him, that of 320 shows a slight difference, while in the other two the difference becomes marked at least in one respect. In all the pipes a note distinctly lower in pitch as well as in intensity than the fundamental makes its appearance at a few millimetres pressure. The minimum pressure to start this note is higher the higher the pitch. This note gets considerably sharper than the fundamental as the pressure rises, and ceases at pressures which are also higher the higher the frequencies. Then in the two lower pipes a note slightly lower than the octave starts a little before or just as the fundamental ceases. In the highest of our set, instead of the octave counterpart a period of silence ensues. This octave counterpart is very feeble in the 320 pipe, and is elicited with difficulty. In the 384 it is produced with considerable difficulty, and ceases abruptly, this being followed by a period of silence. The periods of silence in the two higher pipes are followed by the normal tones, which are, to start with, a bit flatter than the fundamentals. The pressures at which the normal tones commence are also higher for higher pipes. In the lower pipes the octave tones which started as the underblown fundamentals disappear, rise with pressure and cease, giving rise to the normal notes.

Theory and Discussion.

We shall, in the first place, consider the instability of the fluid jet, as this forms the main characteristic on which Helmholtz based his explanation of the initiation and maintenance of vibrations in organ pipes. Lord Rayleigh* obtained the con-

* Theory of Sound, Vol. 2, pp. 377-78.

ditions relating to pressure at the surface of separation of two fluid masses moving with different velocities, the disturbance being supposed to involve x and t through the factors e^{ikx} and e^{int} as

$$g(\rho-\rho^1)+k^2T = k\rho(v+n/k)^2 \coth kl + k\rho^1(v^1+n/k)^2 \coth kl'.$$

He obtained for the criterion of stability

$$(\rho \coth kl+\rho' \coth kl')\{g(\rho-\rho')+Tk^2\}-k\rho\rho' \coth kl \,.\, \coth kl' \,.\, (v-v')^2 > 0$$

From this it follows that when g and T both vanish the motion is unstable for all disturbances, that is, for all values of k. This, with the necessary substitutions, can be easily adapted to the present case of a plane jet of fluid playing into a stationary fluid of its own density.

Both Helmholtz and Lord Rayleigh agree in the statement that the surfaces of the thin blade of air are vortex sheets, and Prof. Tyndall's experiments on burning gas jets establish the fact beyond doubt. The instability of the blade of air might indeed be attributable to the existence of vortices as Helmholtz and Lord Rayleigh had opined, and might, in consequence, easily be conceived to increase with pressure. The vortices further serve as an accelerating force, with periodically alternating direction on the mass of air in the pipe, with the result that the period of the pipe is brought down when their strength increases on stronger blowing. An explanation of the phenomena of overblown pipes may, in fact, be suggested on these lines. The period becomes smaller and smaller for pressures higher than the fundamental pressure until at a particular stage it settles down rather suddenly to that of the octave, the next higher mode of free vibration of the pipe. If, as Lough on the one hand and Bhargava and Ghosh on the other have explained, these vortex pulsations are to be endowed with a free period of their own, this period must perhaps be assumed to undergo transformations similar to the changes of the tone of the pipe as a whole.

Next we have to take into account that the blade, though very thin, has finite thickness, and takes time to travel past the lip as well as the width of the mouth of the pipe. We may suppose that at the fundamental pressure the corresponding width and velocity of the stream are such as to make the time of transit to and fro exactly equal to the time which a compression or rarefaction takes to travel twice the length of the pipe. For higher pressures we know that the velocity and the width of the stream increase, but the former more rapidly than the latter. Now the stream produces a compression or rarefaction a little before the proper phase is reached, and thus reduces the wave-length. This continues until the pipe prefers to jump into the higher mode of free vibration. The variations of the velocity and the width of the stream, each in their own way, render it possible for them to have at other pressures values which make the time of transit of the blade to and fro again equal to the periodic time of the pipe. This happens at the underblown fundamental pressure. Under these circumstances we expect to find here also a counterpart of the phenomena observed in the neighbourhood of the normal fundamental, and this is amply borne out by experiment. The energy supplied being very small, the tones here are necessarily feeble.

In conclusion the authors wish to express their best thanks to Prof. A. L Narayan for his interest and suggestions during the progress of the work, and also to Prof. D. Venkataswamy Naidu, of the Maharajah's Music College, for his assistance in the experimental portion of the work.

DEMONSTRATION.

In the course of his demonstration Mr. BLAIKLEY stated that the upper partials actually given by an organ pipe are not strictly in tune with the notes which would be obtained from the harmonic chord on the fundamental. The pitch of the actual notes is sharper than the pitch calculated from the length of the pipe, owing partly to the correction for the open end, but still more to the correction for imperfect closure at the lip end, the latter correction being different for different partials. By resonance to two tuning forks Mr. Blaikley showed that a pipe having a prime of 256 vibrations by resonance gave a second partial of 530 instead of 512, i.e., a note more than a quarter of a tone sharper than the octave of the prime. The dimensions of this pipe were : Length from upper lip to end, 55·4 cm. ; diameter, 5·28 cm. ; mouth : width, 3·5 cm., height, 0·88 cm. ; slit or flue, 0·15 cm. The blade of air impinging on the lip acts as a reed, and the pitch of the note actually obtained is determined by the interaction between the blade or reed and the pipe ; it cannot be calculated from the natural frequency of one of these elements without reference to the other.

The pipe was next sounded by blowing it steadily with a pump at gradually increasing pressure, the latter being measured with a water manometer. Steady sound was first obtained at 0·28 cm., the pitch being 250, and therefore flatter than the natural fundamental as determined by maximum resonance. At 0·38 cm. the pitch was 259, and the simple tone ceased. At 0·45 cm. the octave sounded with the prime pitch 271, and between 0·45 cm. and 0·81 cm. there was silence. At 0·81 cm. the underblown octave appeared, pitch 516, and at about 1 cm. a critical condition appeared, a mere waft of air over the lip changing the pitch from prime to octave. Between 1 cm. and 1·82 cm. there was silence, and at 2·35 the compound tone 256 sounded, this note gradually rising with increasing pressure. The resonant or natural pitch of the prime being 256 vibrations and the pitch by blowing rising from 250 to 259 with increase of pressure from 0·28 cm. to 0·38 cm., it follows that the true pitch of the pure fundamental is obtained from an intermediate pressure, between 0·28 cm. and 0·38 cm.

With the comparatively high or customary pressures the tone is compound, and the pitch is determined not only by the pressure, but by the presence of the higher proper tones, which are sharper than the harmonic partials of the true resonant prime. The resulting compound tone is therefore in some degree a forced vibration.

In a further experiment two harmonium reeds of about 1,024 vibs. were used, being blown steadily. To one of these a very short length of tube with an adjustable slider was attached, by means of which the varying influence of the tube upon the pitch of the reed was shown. Further, at a certain length of tube sound ceased, but the addition of a quarter, or any odd number of quarter wave-lengths of tube restored speech, whereas with added half wave-lengths silence continued.

DISCUSSION.

Mr. E. G. RICHARDSON referred to the authors' attempt to connect the pressure of the air supply with the width of the air blade and the frequency of the vorticity. The blade is assumed to set up vortices, successive vortices being on opposite sides of it, so that the " frequency of the vorticity " is equal to the quotient of the velocity of the vortices relative to the lip by the distance between successive vortices ; this frequency will, therefore, depend on the velocity of the air blade. A relation of the kind referred to has been obtained by Kruger and his assistants at Greifswald (Annalen der Physik, vol. 60, 1919), who connected the frequency of the note obtained with the velocity of efflux of the air stream and the distance from lip to languid. Earlier experiments on this subject were made by Wachsmüth and Von Schaik in 1905. The speaker had been making similar investigations with a pipe having an annular slit similar to that of the Galton whistle, and had obtained similar results. He suggested that an organ-pipe, when underblown, was responding to a frequency of production of vortices which was a sub-multiple of that of the fundamental of the pipe. He congratulated the authors on their work, and hoped they would be able to put into a more quantitative form the theory outlined at the end of their Paper.

Mr. W. A. BENTON said that the lowest pressure ever actually used in organs was one of $1\frac{1}{2}$ in. found in an early Italian chamber organ ; below such a pressure it would be difficult to get a steady tone. The maximum pressure used in practice is about $3\frac{1}{2}$ in. He suggested that the authors should study a set of pipes all designed to give their fundamentals at the same pressure, e.g., a set forming an organ stop : it would be well if the authors were to mention the height of the mouth and the width of the upper lip in the pipes used by them. The vibration of the metal lip is of small consequence ; it is the air blade that has to be considered.

Mr. W. E. Benton (communicated) : The work described would be of use to other investigators if full particulars of the pipes under examination were given. Of these, the height of the mouth, the depth and breadth of the pipe, the exact position of the upper lip with respect to the wind-sheet, the breadth and form of the slit, the dimensions of the throat and the diameter of the foot are all necessary for any proper interpretation of the results. For instance, if measurements of the mean velocity of the jet for any given pressure had been made with a hot-wire anemometer, it could have been shown that the note given by the pipe is approximately equal to $\frac{iv}{2 \cdot 3a}$ where i is an integer, v is the mean velocity of the jet, and a is the height of the mouth, provided that the breadth of the slit were not more than about 1 mm.

Without such measurements the observations cannot serve to support or contradict any theory whatever.

Lord Rayleigh's criterion of stability is inapplicable here, since V. Hensen's Pitot-tube measurements of the distribution of velocity in a laminar jet proved twenty years ago that there is no surface of discontinuity between a laminar gaseous jet and the surrounding air. That his observations give nearly correct values of the velocity can be shown by integrating for the total momentum across any section, perpendicular to the principal plane of the jet. The work of A. C. Lunn is not applicable to an organ pipe, since the " Schlieren-Bilder " of Prof. Wachsmüth prove that the jet never vibrates in the simple parabolic manner which he assumes, nor does an antinode lie at the mouth of the pipe. R. König showed that in all well constructed open pipes the operation of blowing causes the central node to be depressed by about one fourteenth of the speaking length, causing an appreciable variation of pressure at the mouth.

V. Lough gives too few particulars of his pipes for his results to be of use. The work of S. Bhargava and R. N. Thorp, however, is fairly well detailed, and serves to support the more recent theories of the organ pipe.

The theory given in most treatises on acoustics, concerning the production and maintenance of sound in flue pipes, is incorrect, and Hermann Smith, Cavaillé-Coll, Dr. Audsley and other famous organ-builders have attempted in vain to correct statements which are still upheld by the authority of such names as Helmholtz, Rayleigh, Tyndall and Deschanel.

III.—ON THE TEMPERATURE GRADIENT IN GASES AT VARIOUS PRESSURES.

By W. Mandell, *B.Sc.*, and J. West, *B.Sc.*
(Assistant Lecturers in the University College of Wales, Aberystwyth.)

Received June 26, 1924.

(*Communicated by* Prof. Gwilym Owen.)

Abstract.

A careful re-examination of the problem of the temperature gradients through gases at various pressures has been attempted.

Sets of curves indicating these gradients at various pressures and other interesting relations were obtained. Amongst other features, they show quite definitely the existence in contact with the vessel walls—at low pressures—of very large "temperature jumps."

An attempt to explain the general character of the results is based on the assumption of the presence at the walls, of a gas film possessing certain properties due to the attracting forces existing amongst wall molecules and gas molecules.

Instead of using one thermocouple only to determine the temperature at any point in the enclosure, several thermocouples of differing thicknesses—and therefore differing curvature—were employed under identical conditions. Systematic differences amongst corresponding readings of the thermocouples were noticed. These may also be attributed to the presence round the various thermocouples of gas films whose thickness and character vary with the thickness of the couple.

Introduction.

EARLY experiments on the heat conductivity of gases showed that below a certain small pressure the conductivity no longer remained constant, but diminished with decreasing pressure. Subsequent experiments by Smoluchowski* indicated that this diminution could not be explained on the assumption that the coefficient of conductivity of the gas itself varied with pressure; for, in the case of a thin layer of gas enclosed between two solid surfaces maintained at different temperatures, the effect was found to depend on the thickness of the layer. The obvious conclusion was that the temperature gradient through the gas was not constant.

Smoluchowski suggested that this variation of the conductivity with pressure might be explained if one assumed a discontinuity of temperature at the surface separating solid and gas. The existence of such a discontinuity had previously been predicted on theoretical grounds by Kundt and Warburg.† The phenomenon was supposed to be analogous to the well-known "slip" effect in viscosity; an effect first discussed by Helmholtz for liquids and observed by Kundt and Warburg for gases.

In the case of viscous slip in a gas, the equation representing the state of affairs at the boundary between solid and gas can be expressed thus:

$$\xi\frac{dv}{dn}=v-v'$$

* Wiedemann's Annalen, 64, p. 101 (1898).
† Poggendorf Annalen, 156, p. 177 (1875).

where v =velocity of solid surface.

v' =velocity of fluid at surface.

dv/dn=velocity gradient along normal to solid surface.

and ξ is the coefficient of slip.

In the case of a gas, ξ is found to depend on the mean free path, λ, of the gas molecule. It may therefore be written $=k\lambda$, where k is a constant depending upon the nature of the gas and surface.

Clearly ξ may also be written $=\frac{b\,760}{p}$, where p is the gas pressure in mm.

Analogously—according to Smoluchowski—we have in the case of heat conduction in a gas

$$\gamma\frac{d\theta}{dn}=\theta-\theta' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\gamma=k'\lambda=\frac{a\,760}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The quantity γ is the so-called "temperature jump coefficient," and corresponds to the coefficient of slip in viscosity.

The meaning of equation (1) will perhaps be made more clear by Fig. 1.

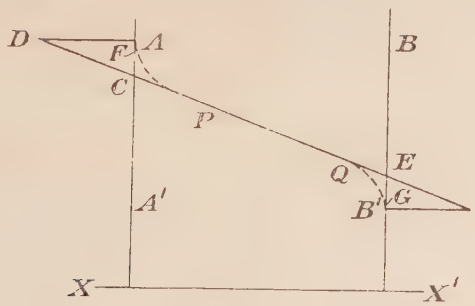


FIG. 1.

AA' and BB' represent the traces of two plane surfaces maintained at temperatures θ_1 and θ_2 $(\theta_1>\theta_2)$, enclosing a quantity of gas. If the heights above XX' represent the temperatures of the plates or gas above some arbitrary temperature, it is clear that the temperature distribution through the gas will be represented by a line on the diagram.

Let the point A represent the temperature θ_1, and B' the temperature θ_2. Then in the simplest case of a uniform gradient through the gas with temperature discontinuities at the plates, the distribution of temperature will be represented by some line $CPQE$. In this case, the slope of PQ—i.e., of CD—is a measure of $d\theta/dn$;

the length AC is a measure of the temperature jump $\theta_1 - \theta'$; and AD is a measure of the coefficient γ.

Several investigations have been carried out with a view to testing this theory, relating to the discontinuity of temperature between a solid and a gas ; and, in some cases, to finding the value of the product γp, or virtually of the constant a of equation (2).

Amongst the investigators may be mentioned Kundt and Warburg,* Smoluchowski,† Knudsen,‡ and Gehrcke.§ The most direct method, however, was that used by Lasareff,‖ who attempted to measure the actual temperature gradient through a gas layer contained between two parallel metal plates maintained at different temperatures.

The temperature differences employed by him—as by most of the workers previous to him—were small, the largest being about 20°C. Moreover, the temperature of the metal surfaces in contact with the gas were assumed to be those of the oil baths used to heat or cool them.

Of recent years, the experiments of Langmuir¶ and others have brought into prominence the view that a solid surface is in general covered with a film of adhering gas molecules. It is natural to suppose that the temperature gradient in the neighbourhood of the plates in the case referred to above will be affected by the presence of such a film. If this be so, the temperature distribution between the plates in Fig. 1 will not be represented by the straight line $CPQE$, but more probably by some curve $FPQG$, where F and G are points between C and A and E and B' respectively. The discontinuity or temperature jump between gas and plate in the simple case first considered and represented by AC, will now refer to the gas *proper* and the plate ; we may regard the gas *film* as acting somewhat in the manner of a transition layer.

It seemed probable that information on these various points might be obtained by a study of the temperature gradients through different gases in an unequally heated enclosure at different pressures. Such a study might also yield information as to the properties and character of the gas films when present. It was primarily with these objects in view that the experiments described in this Paper were undertaken.

Assuming a connection to exist between the above-mentioned gas film and the temperature jump in the neighbourhood of a solid surface, it is clear that the problem to be studied may become somewhat complicated ; for such gas films must also be present at the surfaces of the thermometers used to investigate the temperature gradient—in our case thin wire thermocouples—and difficulties may arise as to the correct interpretation of the various indications. Further, it is not unreasonable to suppose that, in addition to such obvious possible factors as the nature and temperature of the gas and solid, and the pressure of the gas, the properties of the film may to some extent depend upon the curvature of the solid surface. This possible complication led us to employ for the examination of the temperature gradient several thermocouples of different thicknesses instead of one only.

* Poggendorf Annalen, 156, p. 177 (1875).
† Wiedemann's Annalen, 64, p. 101 (1898).
‡ Annalen der Physik, 34, p. 593 (1911).
§ Annalen der Physik, 2, p. 102 (1900).
‖ Annalen der Physik, 37, p. 233 (1912).
¶ Physical Review, 43, p. 149 (1916).

By reapproaching the whole problem along the above lines some rather interesting results have been obtained. Those relating particularly to the temperature gradient through a gas are dealt with in this Paper, whilst those relating to the effect of curvature on the indications of a thermocouple will be treated more fully in a subsequent Paper.

Apparatus.

Fig. 2 represents diagrammatically a section of the chief portion of the apparatus.

A cylindrical brass tube of diameter 12 cm. and of length 3·9 cm., was sup-

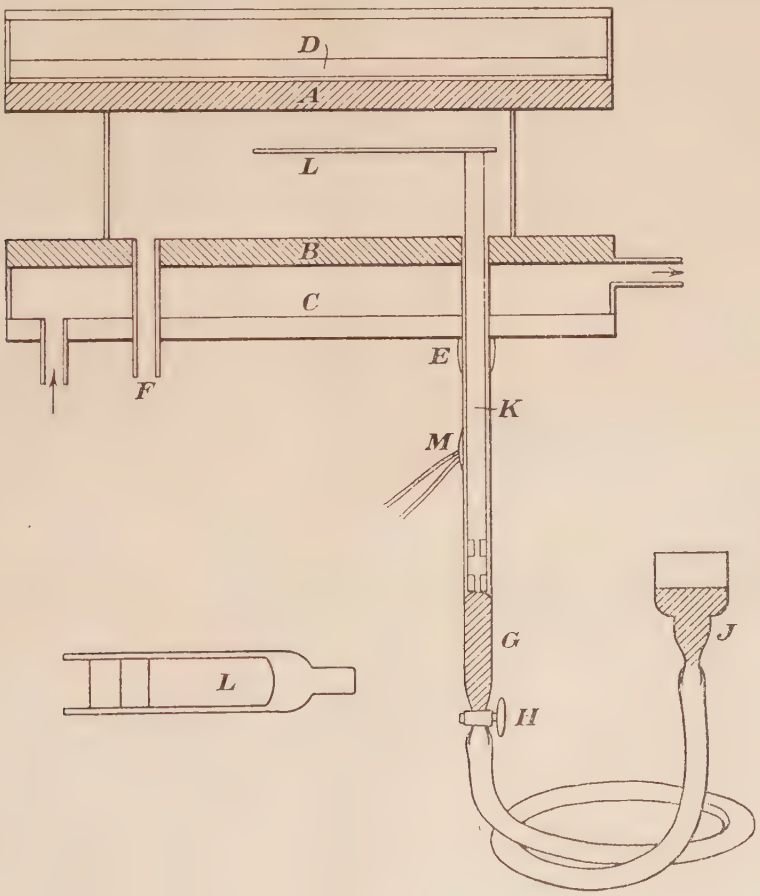


Fig. 2.

ported with its axis vertical. The ends of the tube were closed by two carefully turned and machine-faced brass discs of larger diameter, 18 cm., and of thickness 0·5 cm.

The upper disc A, referred to subsequently as the "hot plate," formed the lower surface of a brass box which contained an electrically heated coil D. This coil—a flat spiral of nichrome wire—covered the whole of the plate A, and thus, extending beyond the circular walls of the test-chamber, ensured a uniform temperature over that part of A which formed a wall to the enclosure. The heating coil was supplied from a 100-volt battery; it was seldom found necessary to adjust the current after an experiment had begun, the temperature of the hot plate remaining very steady for several hours at a time.

The lower plate B—the "cold plate"—formed part of a brass tank C, through which a steady and controllable stream of cold water flowing under constant head was passed. As in the case of the heating arrangement, and for a similar purpose, the cooling apparatus extended beyond the circular walls of the test chamber. In this case also the temperature of the cold plate usually remained constant throughout an experiment.

Into two small holes cut in the cold plate, on opposite sides of the centre and near the edge, were soldered two brass tubes E and F, which passed through the water tank C to the outside. The tube F was connected to the supply of gas under investigation, and to the pumps and McLeod pressure gauge. The vertical tube E was connected permanently to a vertical glass tube G, furnished with a stopcock H, which permitted communication with a mercury reservoir J, whose height could be varied. It was thus possible by suitably manipulating J and H to adjust delicately the mercury level in G to any required height.

Floating in the mercury in G was a thin quartz tube K, which penetrated into the experimental chamber. The upper end carried a very thin horizontal porcelain platform L, of which the dimensions and plan are given in inset to Fig. 2. On this platform, and in the same plane, were mounted three independent thermocouples of different diameters.* These thermocouples, which were of copper and constantan, and of diameters 0·0044, 0·0126 and 0·0209 cm., were made so as to have practically no blob at the junctions. They were stretched across the arms of L, as in Fig. 2, about 5 mm.s apart, and were fixed to the platform with an insulating cement; their general appearance was that of three almost accurate coplanar parallel cylinders of common length 1·2 cm. The wires, except the length just mentioned, were completely covered with a thin layer of cement, and passing down the quartz tube left the apparatus through a hole M in the glass tube G.

In addition to the above thermocouples, two more were used; one was soldered to the hot plate, the other to the cold plate, at about 1·5 cm. from their centres. The wires from these couples passed through the tube F to the outside.

Before the five thermocouples were actually fixed in the apparatus, they were roughly calibrated relatively, and against a mercury-in-glass thermometer in a black-body enclosure. They were again more accurately calibrated in the same enclosure at the end of the experiments. The fact that the various results obtained over many months show no variations whatever, justifies the adoption of this final calibration for application to the whole series of results; particularly to those given in this Paper, which were obtained within one month of such calibration.

* Owing to an accident later, only two of these three couples were available. These are referred to in the text as the *thin* (0·0126 cm.) and the *thick* (0·0209 cm.) couples respectively.

The method of determining the distance of the plane of the platform thermocouples from the hot and cold plates was as follows :—

TABLE I.

Air in chamber. Pressure: atmospheric. Hot plate temperature =179°C. Cold plate temperature = 49·5°C.		Results for thin thermocouple. Pressure=0·0015 mm. Hot plate temperature =179·3°C. Cold plate temperature = 49·0°C.	
Distance from *H*.	Temperature.	Distance from *H*.	Temperature.
mm.	°C.	mm.	°C.
1·00	163·3	1·00	108·0
1·95	152·2	2·50	102·7
2·95	143·2	3·50	101·4
5·05	131·5	7·50	98·2
7·20	125·1	15·55	95·6
13·75	109·5	26·60	90·0
23·35	93·5	31·25	87·3
33·50	76·3	33·00	86·3
35·50	70·1	35·50	85·0
37·05	63·0	37·00	83·3
37·90	57·0		

TABLE II.

Hydrogen in chamber. Pressure: atmospheric. Hot plate temperature =171·5°C. Cold plate temperature = 47·7°C.		Results for thin thermocouple. Pressure=0·0027 mm. Temperature of *H*=171·2°C. Temperature of *C* = 48·5°C.	
Distance from *H*.	Temperature.	Distance from *H*.	Temperature.
mm.	°C.	mm.	°C.
1·00	164·7	1·00	111·8
1·80	160·4	2·00	109·7
2·80	155·9	3·15	107·5
4·35	149·2	5·80	106·4
7·25	138·2	13·00	103·9
9·80	129·9	20·80	101·8
15·05	115·5	27·05	100·0
19·65	103·4	34·10	98·2
26·65	85·0	36·00	96·7
29·80	76·5	36·90	95·7
33·55	67·0		
35·00	62·7		
35·80	59·2		
37·00	56·0		

Before the cylinder and hot plate (previously soldered together) were placed in position on the cold plate, the height of the thermocouples above the centre of the cold plate was found by means of a cathetometer by observing the heights of these couples above a series of points on the rim of the cold plate which was horizontal. At the same time, by means of a vertically travelling microscope, the height of a fiducial line on the quartz tube as seen through the glass tube *G* was found. The microscope readings were thus calibrated against those of the cathetometer. In the

same way, when the hot plate and cylinder had been soldered into position, observations by the cathetometer on points of the rim of A gave the height of the centre of the hot plate. The error in the estimation of the distances of the couples from the two plates is probably not greater than 0·1 mm.

In the absence of a more convenient apparatus, the following potentiometer arrangement was used to measure the E.M.F.s of the thermocouples :—

E is an accumulator.
W is a standard cadmium cell (1·0184 volts).
M is a variable resistance.
G is a low resistance galvanometer.
SR is a subdivided megohm to the mid-point of which the cell W is connected.
P, Q are two calibrated resistance boxes.

The resistance $P+Q$ was kept constant and equal to 9,200 ohms, whence it follows that for no deflection of the galvanometer G, when the points a and c or b and c are joined, the resistance in ohms taken from the box Q is equal numerically to one-half the E.M.F. of the thermocouple under test in microvolts. A common cold couple was kept at room temperature in a water bath.

With this arrangement, when dealing with temperatures giving E.M.F.s of the

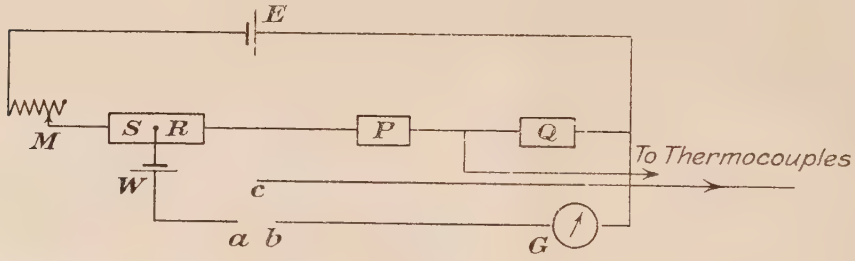


Fig. 2a.

order of 8,000 microvolts, readable deflections were obtained for changes of four microvolts. The manipulation of the resistances was somewhat laborious, but the results obtained were very consistent.

It will be seen from the foregoing description, that we have in effect a layer of gas some four centimetres thick, enclosed between two horizontal plates—maintained at different temperatures—under conditions in which convection, and the influence of the sides on the main body of the gas may be assumed negligible ; and in which the lines of flow of heat—at any rate through the central portion of the layer—may be assumed vertical. That these assumptions were justified was shown by swinging the thermocouples round the axis of the quartz tube in a horizontal plane when the temperatures indicated were found to be constant.

Method of Carrying out an Experiment.

In an experiment, say, with air, the apparatus was many times alternately exhausted, and filled with air dried by passing over calcium chloride and phosphorus pentoxide. Finally, the apparatus was filled with dry air at the desired pressure.

During this time, the hot and cold plates were taking up the temperatures

chosen. Depending upon the circumstances, usually from three to four hours elapsed before conditions were steady, and observations could be commenced.

The platform thermocouples which had previously been placed as close to the hot plate as possible, were then gradually moved downwards by measured steps, until the cold plate was reached. At each step, the E.M.F.s registered by all the thermocouples, including those on the hot and cold plates, were noted. These observations usually took about two or three hours, since for each movement of the platform, it was necessary to await steady conditions.

For the sake of uniformity and ease of comparison, the experiments described in this Paper were carried out with the hot and cold plates at constant temperatures of about 180°C. and 50°C. respectively.

Results.

With the exception of those given in the Appendix, the results in this Paper deal with the general problem of the temperature gradient through a gas.

The results given in the Appendix show that under certain conditions—e.g., in air at atmospheric pressure—there is a systematic difference between the readings of the thick and thin thermocouples. Presumably on all grounds the results obtained with a very thin thermocouple will most nearly represent the true state of affairs in the gas chamber. For this reason, and for the sake of uniformity, the graphs and calculations have been based on the indications of the thinner thermocouple, although of course, readings were taken for both.

An examination of the results given in the Appendix will show that the greatest difference between the readings of the two couples was only about $1\frac{1}{2}$°C. Thus, although it is possible that with still thinner thermocouples the difference might be larger it is probable that the present results represent with a fair degree of accuracy the temperature distribution through the gas.

In the tables the temperatures are expressed in degrees centigrade. They have been deduced from the calibration curve which was obtained, as previously explained, in the black body enclosure.

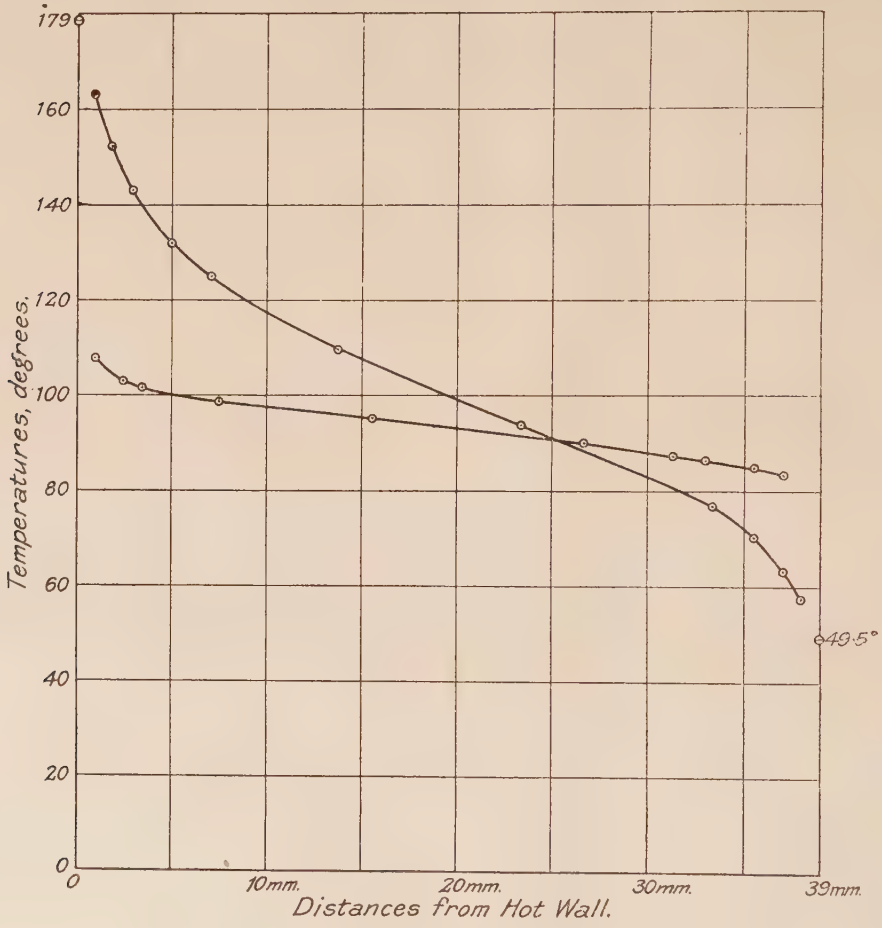
The graphs have been constructed directly from the tables given. It was not possible for the platform thermocouples to approach the plates more closely than 1 mm., and no attempt has been made to extrapolate the experimental curves. It may be observed, however, that in the curves for very low pressures (*see* Graphs 1 and 2) those curves appear, if produced, to cut into the plates at points not coincident with those representing the temperatures of the plates ; thus indicating the existence of a discontinuity of temperature at the plate in addition to a variable gradient in the neighbourhood. This is not noticed in the case of high pressures.

The upper curve in Graph 1 shows the temperature distribution across the chamber with air at atmospheric pressure. The hot plate (H) had a temperature of 179°C., and the cold plate (C) one of 49°C. It is seen that the temperature distribution is not uniform. In the first 5 mm. from H, the temperature falls 46°C., in the last 5 mm. it falls 26°C., whilst across the remaining 29 mm. the drop is only 58°C.

The form and general dimensions of the chamber ensure that approximately the same quantity of heat per second crosses every central horizontal section of the chamber—i.e., $dQ/dt = K d\theta/dn$ = constant, where K is the conductivity coefficient, and $d\theta/dn$ is the temperature gradient along the vertical axis. The results therefore appear to indicate a variation in the conductivity coefficient K in the regions near H

and C; which suggests the presence of some sort of transition layer of air near the plates.

The lower curve in Graph 1 gives the temperature distribution with air at a pressure of 0·0015 mm., the temperatures of the plates being the same as for the upper curve. We now have a small uniform gradient through the bulk of the gas (hereafter referred to as the *bulk gradient*), with large jumps at the ends. At a distance of

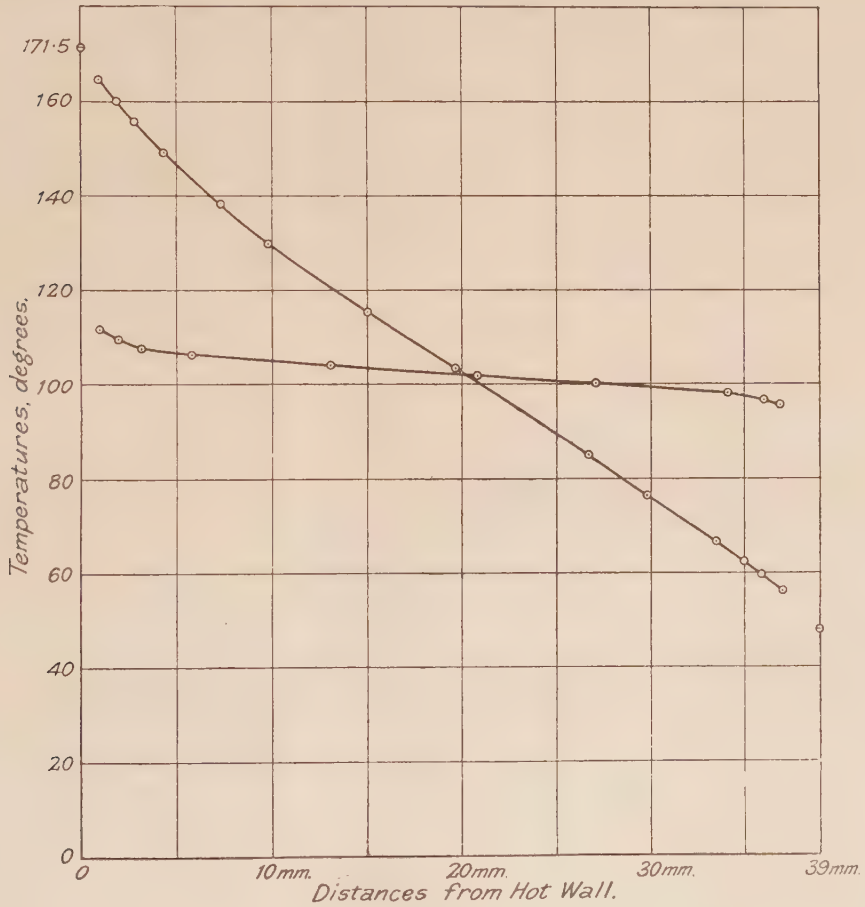


Graph 1.

1 mm. from the hot plate, there is a fall of 71°C. If the straight portion of the curve be produced both ways to meet the plates H and C, we get temperature jumps of 77°C. at H and 35°C. at C, leaving only 18°C. for the fall across the chamber. It is clear that the character of the transition layer is in some way different from that present at atmospheric pressure.

Graph 2 indicates the results when hydrogen from a gas cylinder replaced the air in the chamber. The upper curve refers to hydrogen at atmospheric pressure,

the lower to the same gas at a pressure of 0·0027 mm. Although both curves are of similar type to those in Graph 1 for air, certain differences are to be noticed. In the upper hydrogen curve, the bulk gradient is seen to be more nearly uniform and steeper than in the corresponding air curve. Thus, in the first 5 mm. the temperature drop is only 24°C., in the last 5 mm. it is 18°C., whilst across the remaining 29 mm. it is 81°C. In the lower hydrogen curve, the bulk gradient is uniform, but less steep than in the corresponding air curve. Further, whilst in the case of air at low pressure the tem-



Graph 2.

perature jumps at the two plates are very much different, the corresponding jumps in the case of hydrogen are much more nearly alike. The same effect, but less noticeable, can be found in the upper curves of Graphs 1 and 2.

The comparison may be summed up in the statement that for the same temperatures of hot and cold plates, the bulk gradient in hydrogen for various pressures varies over wider limits than that in air, whilst the general temperature distribution between the plates is more nearly symmetrical in the case of hydrogen.

Table III was compiled in a somewhat different manner from the preceding tables.

TABLE III.—*Air.*

Temperature of hot plate = 173·7°C.
Temperature of cold plate = 49·0°C.

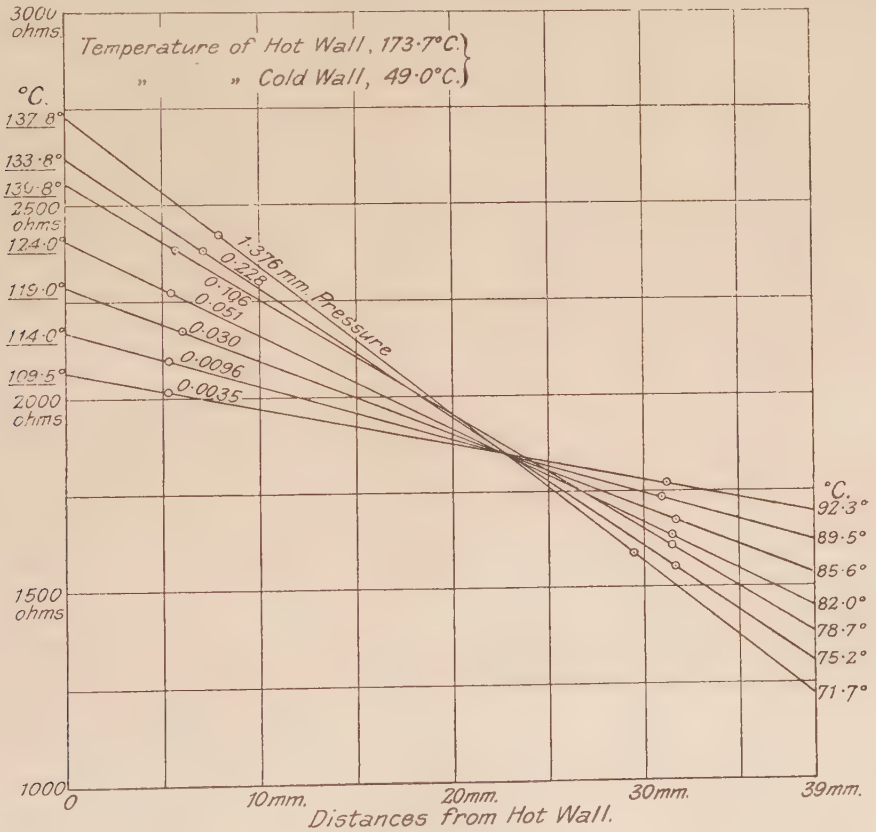
Pressure.	Distance from hot plate.	Temperature in terms of ohms out of box Q.	Temperatures at H and C in deg. C. on producing lines in Graph 3 to meet plates.	
			θ'_H.	θ'_C.
mm.	mm.	ohms.		
1·376	7·85	2,427	137·8	71·7
	29·60	1,587		
0·228	7·00	2,388	133·8	75·2
	31·40	1,557		
0·106	5·60	2,390	130·8	78·7
	31·25	1,613		
0·051	5·40	2,274	124·0	82·0
	31·25	1,634		
0·030	5·90	2,176	119·0	85·6
	31·80	1,683		
0·0096	5·30	2,096	114·0	89·5
	30·75	1,739		
0·0035	5·30	2,019	109·5	92·3
	31·00	1,768		

Several curves at different low pressures in air—of which only one is given in Graph 1—showed quite clearly that the bulk gradient remained perfectly uniform until within about 3 mm. of the hot and cold plates. This fact suggested that, in order to determine the bulk gradients, and hence the value of $d\theta/dn$ at low pressures, it was only necessary to determine the temperatures of the thermocouples for two positions of the platform, one about 4 mm. from H, the other about the same distance from C. It would thus be possible to obtain in a comparatively short time—several hours instead of several days—and therefore under the same conditions, the bulk gradients corresponding to different low pressures—information of considerable interest. This was done, and the results are embodied in Table III. and in Graph 3.

The maximum possible bulk gradient is clearly $(\theta_H - \theta_C)/HC$, where HC is the distance between the hot and cold plates. Although from very low pressures and up to about 0·06 mm. the gradient is seen rapidly to increase, there is but little change onwards to 0·2 mm., and from 1·5 mm. to atmospheric—the highest pressure used—the gradient appears to remain stationary.

The fact that all the lines pass through one point suggests that whatever the actual temperature jumps at the plates may be, the *ratio* of the hot "jump" to the cold "jump" for fixed temperatures of the two plates is independent of the pressure. This ratio is that of the distances of the point of intersection of the lines from the two plates, and is seen to be as $45\frac{1}{2}$ to $32\frac{1}{2}$. It is interesting to notice that this ratio is the same as that of the absolute temperatures of the hot and cold plates, viz., 447 to 322. Unless this be a curious coincidence, it suggests that the temperature jumps at the plates for air for any fixed pressure are proportional to their absolute temperatures.

The variation of the bulk gradient—the $d\theta/dn$ of equation (1)—with pressure is well brought out in Graph 4, which was constructed from the same data as Graph 3. The curve stops at the point corresponding to the gradient for the lowest pressure used, and no attempt has been made to estimate the non-experimental portion. Above 0·06 mm., $d\theta/dn$ slowly increases with pressure, apparently reaching a final and maximum value at about 1·5 mm. Below 0·06 mm., $d\theta/dn$ diminishes very rapidly indeed, and the curve, if produced, would apparently pass through the origin. Such an occurrence could only indicate one of two improbable things.

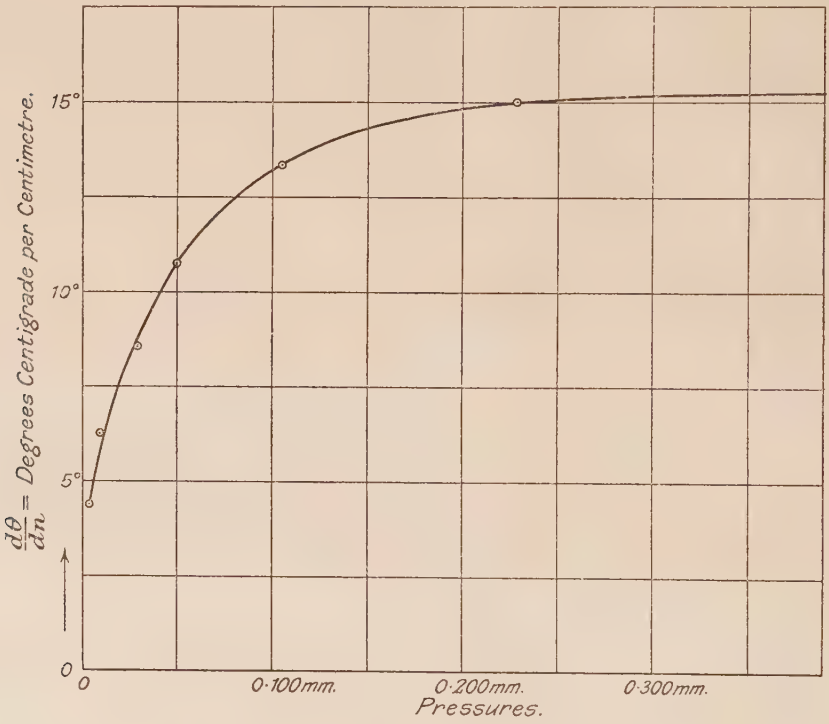


GRAPH 3.

Either the gradient in a perfect vacuum under the above conditions is zero—a difficult fact to conceive if the couples receive any heat due to radiation—or the radiating surfaces are not the metal plates, but some other surfaces. In the latter case, we may assume them to be the outer surfaces of the gas films adhering to the plates, and must then assume the temperature drops through these films to be such that these two surfaces have equal temperatures. We can be almost certain, therefore, that the curve cuts the gradient axis at some point near to, but not coincident with, the origin, indicating that though finite, the effect due to radiation is very small.

It is interesting to examine Graph 4 in conjunction with some results obtained by Soddy and Berry* when investigating the heat losses from a hot wire through gases at different pressures. They obtained for the variation of the heat loss per second with pressure a curve almost identical with the one given in Graph 4. It is clear that in the equation $dQ/dt = K d\theta/dn$, Soddy and Berry found the variation of the quantity dQ/dt with pressure, whilst the experiments described in this Paper give the variation of $d\theta/dn$ with pressure.

It is true that the two sets of experiments were carried out under very different conditions; but if we assume for the moment that the variation of the bulk gradient is the same in both cases, we see that the quantity K is really a constant. In any



GRAPH 4.

case, a method of experimentally finding the variation of K with pressure is suggested; for it is simply required to investigate at one and the same time, i.e., under the same conditions, the variation with pressure of both dQ/dt and $d\theta/dn$. The apparatus used in this Paper is inconvenient and ill adapted for such an experiment, but one similar to that used by Soddy and Berry would, with the necessary modifications, be very suitable.

Comparing once more Graph 3 with Fig. 1, it is seen that the $\theta_H - \theta^1_H$ of equation (1)—which is the temperature jump at the hot plate—may be obtained by producing back the lines to meet that plate. Incidentally, the intersection of the same lines

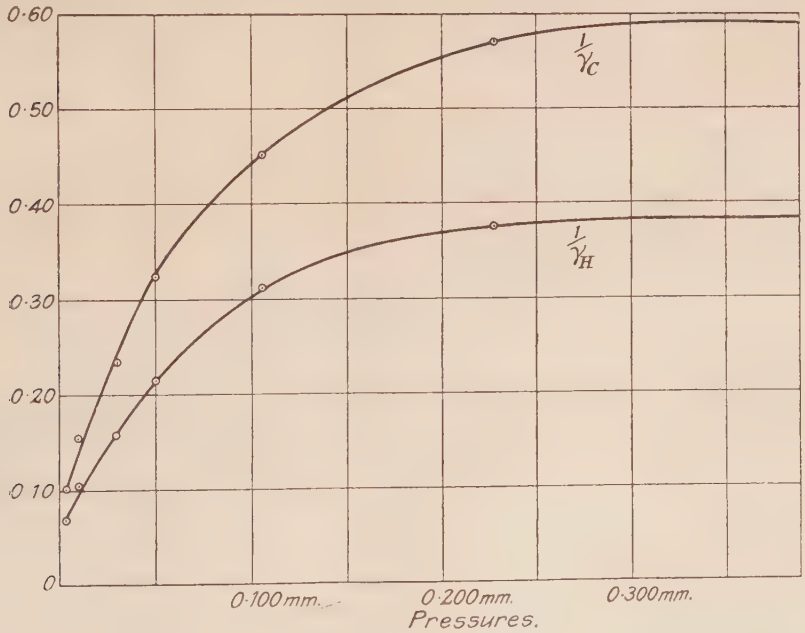
* Proc. Roy. Soc., A., 83, Dec. (1909), Nov. (1910).

with the cold plate serve to determine $\theta_C - \theta^1_C$. The results of such operations are collected in columns 3 and 4 of Table IV. From the data of columns 2 and 3 it is now possible to determine γ_H as defined in equation (1). For convenience, the values of $1/\gamma_H$ and $1/\gamma_C$ are given in the table.

TABLE IV.—*Air.*

Pressure.	$d\theta/dn$.	$\theta_H - \theta^1_H$	$\theta_C - \theta^1_C$	$\frac{1}{\gamma_H} = \frac{d\theta/dn}{\theta_H - \theta^1_H}$	$a_H = \frac{p\gamma_H}{760}$	$\frac{1}{\gamma_C}$	a_C.
mm.	°C. per cm.	°C.					
1·376	16·95	35·9	22·7	0·472	0·003830	0·747	0·00242
1·228	15·00	39·9	26·2	0·376	0·000798	0·572	0·000524
1·106	13·36	42·9	29·7	0·311	0·000448	0·450	0·000310
0·051	10·77	49·7	33·0	0·217	0·000310	0·326	0·000206
0·030	8·56	54·7	36·6	0·156	0·000252	0·234	0·000169
0·0096	6·28	59·7	40·5	0·105	0·000120	0·157	0·000080
0·0035	4·41	64·2	43·3	0·069	0·000067	0·102	0·000045

The lower curve in Graph 5 shows the variation of $1/\gamma_H$ with pressure. Accord-



GRAPH 5.

ing to equation (2), this graph should be a straight line. The graph convincingly indicates, however, that $1/\gamma$ is not a linear function of the pressure; indeed, an equation of an exponential form would appear more nearly to express the results.

The same type of curve (the upper one in Graph 5) is obtained for the variation of $1/\gamma_C$ with pressure, as is to be expected.

These curves are very instructive; they seem to indicate that the analogy between the temperature jump effect in conduction and the slipping effect in viscosity

is not strictly true. It is, however, quite possible that the velocity gradient in the neighbourhood of a solid surface in the case of viscosity would, if examined by a direct method, show a similar variation with pressure.

It will be seen from column 6 of Table IV. that the quantity a diminishes with decreasing pressure; this is otherwise obvious from Graph 5. The values obtained by Smoluchowski, who believed a to be really constant, was 0·000017 for air in contact with a glass surface.* Lasareff,† working with nickel surfaces and air, obtained values ranging from 0·000013 to 0·00002. The latter's results show some irregularity, some of the differences being of the order of 40 per cent. of the mean value adopted On the other hand, it will be noticed that the values of a given in Table IV. show a *systematic* variation. It will be further noticed that as the pressure decreases, the value more nearly approaches that of other workers, and an examination of Graph 5 shows that their values will ultimately be reached at sufficiently low pressures. By comparing a_H and a_C, as given in columns 6 and 8 of Table IV, it appears that the value of a diminishes with temperature. In this connection it is interesting to notice that Lasareff and most of the previous investigators worked with temperatures of the order of 15°C. This may assist in explaining the differences in the results given in this Paper from those quoted. It would be interesting to examine the variation of the quantity a with temperature when the pressure is fixed.

General Considerations and Discussion of Results.

Consider an experimental chamber of ideal type, in which the hot and cold plates are two infinite plane parallel surfaces H and C respectively, maintained at temperatures θ_H and θ_C ($\theta_H > \theta_C$); and suppose at first that no gas films are present at the plates.

Case 1.—Let the pressure be so small that only a few molecules are present, and no mutual collisions occur. In this case, assuming a perfect interchange of energy at the plate, molecules will leave H with an energy E_H corresponding to θ_H. The same number will leave C with an energy E_C corresponding to a temperature θ_C. Now, since there are no mutual collisions between the molecules, it is clear that the mean energy of the molecules at any point of the chamber between H and C will be constant and $=\frac{1}{2}(E_H+E_C)$. If, therefore, a suitable thermometer could be introduced into the chamber, it would—neglecting any effects due to radiation—register a temperature $\frac{1}{2}(\theta_H+\theta_C)$ along HC. In this case we should have a zero gradient, as in Fig. 3.

The fact that even at relatively high pressures (0·001 mm.) the experimental gradient approaches this in type (*see* Graph 1) shows that the effect due to radiation must be very small, so that in the above case the gradient would differ very little from zero.

Further, we see that although the gradient through the gas may approach zero, a definite amount of heat has been transferred from H to C. It is obvious, and to be expected, that the usual relation for heat conduction $\frac{dQ}{dt}=K\frac{d\theta}{dn}$ cannot now hold. We must now regard the heat as transferred rather by a sort of convec-

* Wiedemann's Annalen, 64, p. 101 (1898).
† Annalen der Physik, 37, p. 233 (1912).

tion than by conduction, as usually understood. Under these circumstances the state of affairs would be represented by an equation of form

$$\frac{dQ}{dt}=K^1 n\,(\theta_H-\theta_C),$$

where n is the number of molecules per unit volume, and K^1 is some constant for the particular gas under consideration.

Case 2.—Let now the gas pressure be increased so that very frequent collisions occur between the molecules. It is clear that if we consider a region near H, although the molecules leaving H will possess a mean energy E_H as before, those approaching H will not have come from C; they will have suffered many collisions, and in consequence statistically their energy will be greater than E_C. Thus the mean energy of the molecules in the region considered will correspond to a temperature $>\frac{1}{2}(\theta_H+\theta_C)$, the degree of inequality depending upon the distance from H. Similarly, near C, the temperature will be $<\frac{1}{2}(\theta_H+\theta_C)$. We thus now get a finite temperature gradient along HC, the magnitude depending upon the pressure.

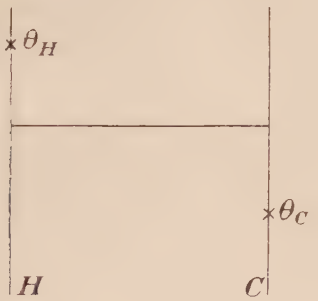


Fig. 3.

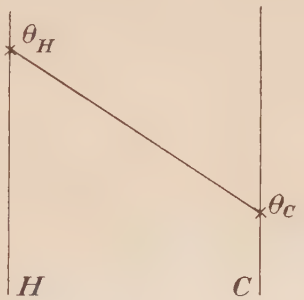


Fig. 4.

The character of the transfer of heat will, when the principles of the ordinary kinetic theory apply, be represented by the usual equation

$$\frac{dQ}{dt}=K\frac{d\theta}{dn}$$

Between Cases 1 and 2 there will, of course, be a transition stage, during which it becomes difficult to associate the coefficient of heat transfer with the coefficient of conductivity as usually defined.

Such elementary considerations would lead to the conception of a gradient somewhat similar to that in Fig. 4, when the pressure is such that the mean free path of the gas molecule is exceedingly small compared with HC, the distance between the plates.

The experimental gradients at pressures satisfying this condition, however, are distinctly different (cf. Graphs 1 and 2). A further study of Graphs 1 and 2 shows that the temperature jumps at the two plates are not the same.

Obviously, therefore, we are dealing with some phenomenon which is peculiar to the region at the wall. It is therefore necessary to examine the mechanism by which the heat transfer between wall and gas occurs. To attempt an analytical

discussion of an exceedingly difficult and complicated problem in a Paper of this kind, where the experimental data are ill adapted for numerical verification, would be out of place ; the object in this Paper is merely to suggest in a general manner one explanation of the results. The treatment is inspired by a very suggestive Paper by Langmuir,* in which, in addition to a summary of previous attempts to explain what occurs at the surface of contact between a plate and a gas, he gives a detailed discussion of the general problem.

Proceeding along lines suggested by Baule,† Langmuir attempts to take into account the attractive and repulsive forces which must undoubtedly exist between the molecules of the plate and gas. By doing so, he is able to account for molecules leaving a hot surface with an energy which may be considerably less than that corresponding to the temperature of the surface. However, the effects of these mutual molecular forces do not appear to have been followed to their logical conclusion.

Consider once more the hot plate H, which may be regarded as composed of molecules arranged in some regular configuration. These molecules will vibrate about their positions of equilibrium with a mean kinetic energy dependent upon θ_H and with a probable amplitude not greater than their diameters. According to Langmuir's analysis, a gas molecule approaching the hot plate is subjected to an attractive force exerted by a group of plate molecules, but is finally repelled by a single molecule only ; if such a molecule return to the gas, it will do so with a kinetic energy greater than its former energy, but less than that of the plate molecules. For our purpose, we may say that for a single collision the impact may be regarded as imperfect.

Consider, now, a gas molecule approaching H with an energy E_0, corresponding to some temperature θ_0 where $\theta_0 < \theta_H$. When close to H, it will, as just stated, be subjected to an attractive force exerted by the group of plate molecules near it and will proceed towards the plate with increasing velocity. On reaching the plate, one of three things may occur.

It may (*a*) strike a plate molecule and be sent back into the gas ; (*b*) strike a plate molecule, but not return to the gas immediately ; or (*c*) not collide with a surface molecule as in (*a*) and (*b*), but pass through the group and enter the solid.

In Case (*a*) of one collision, the gas molecule will re-enter the gas with an energy E_1 greater than its previous energy, but less than E_H. Further, one may reasonably assume that the direction after impact will depend to some extent upon that before. We may therefore regard such a molecule as having suffered reflection at the plate.

In Case (*b*), the gas molecule may, whilst it remains within the region of molecular attraction, suffer a number of collisions, thus moving irregularly over the surface of the plate ; it may also vibrate about some position of temporary equilibrium. It may then re-enter the gas, or it may enter the solid as in Case (*c*). It is clear that such molecules will have a better chance of acquiring an energy E_H than those of type (*a*).

In Case (*c*), the molecule will in the mean suffer so many collisions that it will certainly finally possess energy E_H, corresponding to a temperature θ_H.

When the molecules of the two last types finally return to the gas, although

* Physical Review, 43, p. 149 (1916).
† Physical Review, 43, p. 149 (1916).

he (c) molecules certainly, and the (b) molecules possibly, will start from the plate vith energy E_H, they will not enter the gas with it; for they will lose energy in vercoming the attractive forces which exist at the surface of the plate. Further, he directions of these molecules after leaving the plate will be quite independent f the previous directions of approach. Such molecules are not therefore reflected, ut must be regarded as having first condensed upon, and then evaporated from, he surface.

The general situation at the hot plate may be expressed thus :—

Let E_H=mean energy of plate molecules at temperature θ_H.

E_0=mean energy of incident gas molecules.

E_1=mean energy of escaping gas molecules of type (a).

E_2, E_3=mean energy of escaping gas molecules of types (b), (c).

a, b, c=respective proportions of molecules of types (a), (b), (c).

F_H=mean energy of all gas molecules at hot plate. It will be a measure of the true temperature at this point.

Then

$$F_H=\frac{E_0+(aE_1+bE_2+cE_3)}{2}$$

The values of a, b, and c will depend upon several considerations even when the pressure and temperature are fixed. Evidence from other sources as to the value of a is very conflicting (*see* cases quoted by Langmuir*), but the balance is perhaps in favour of the belief that at low pressures for all gases except hydrogen it is almost zero. In the above equation which refers to the hot plate, E_1, E_2, and E_3 are separately greater than E_0 and less than E_H.

There will be a corresponding equation for the cold plate, viz.,

$$F_C=\frac{E'_0+(a'E'_1+b'E'_2+c'E'_3)}{2}$$

The effects of the various terms will not, however, be in the same direction in both equations; for whilst, for example, E'_1 will have a value intermediate between E'_0 and E_C, the value of E_3 must always be less than E_C. The value of E'_2 may be either $>$ or $<$ E_C, depending upon the circumstances. This may explain the lack of symmetry in the curves of Graphs 1 and 2 to which attention has already been directed.

Finally, whilst E_1 is dependent upon both E_0 and E_H, possibly in the manner discussed by Langmuir, E_3 will be equal to $E_H-\varepsilon$, where ε is the work done by the escaping molecule in overcoming the attractive forces at the plate. The value of E_2 will have some value between E_0 and $E_H-\varepsilon$. The case of the cold plate is obviously similar.

We may regard the three types (a), (b), and (c) as representing the gas molecules which are (a) reflected, (b) condensed upon and (c) absorbed by the surface. Their relative proportions will depend upon a variety of circumstances. Thus, if a gas be gradually introduced into a perfectly evacuated vessel whose walls are originally

* Physical Review, 43, p. 149 (1916).

gas-free, we may be certain that at first a large proportion of the molecules will belong to type (*c*). In fact, we may say that at all pressures and temperatures there is an attempt to saturate the solid and to form a gas layer upon the surface. The effect of increase of pressure must be to assist this action, that of temperature in general to oppose it. When, in the case just quoted, for a given temperature the pressure is such that we may regard the interstices of the solid near the surface as in the mean full, and a film of molecules—at first, perhaps, only one deep—to be on the surface, we shall have a new state of affairs. The number of molecules of type (*c*) will be small, and those of types (*a*) and (*b*) will now collide in general not with the wall molecules, but with the gas molecules in the film. The magnitude of the attractive forces considered above will be different from and less than formerly. Because of this, and also because molecules in the gas film will be less firmly held together than the surface wall molecules themselves, the degree of interchange of energy will be greater.

Just as a film of gas molecules forms on the surface of the solid, so a second set of gas molecules will form on the first set, and so on. But it is clear that succeeding sets will be less firmly held together than the preceding ones, with the result that, although as the pressure increases the film will thicken, it will not be uniformly dense, but will merge finally within a short distance into the gas proper. The effect on the degree of interchange of energy in collisions is obvious; the collisions of gas molecules from the main body of the gas with those in the outermost layer of the film will be almost identical with collisions amongst the free molecules themselves. As the gas layers merge into the gas proper, the conception of a surface disappears, and it is clear that molecules of type (*b*) will gradually become of type (*c*), which, entering the gas film, will move about in very weak fields, and when escaping will only have to do so against very weak attractive forces.

Applying this conception of a permanent gas film at the plates to the experimental results, it is clear that the bulk gradient will depend on the temperatures of the outer layers of the gas films at the two plates, i.e., on the thickness and character of the films.

Graph 4 for air shows that from zero pressure to about 0·06 mm. the bulk gradient $d\theta/dn$ very rapidly increases, but that the *rate* of increase quickly falls off from 0·20 mm. to about 1·5 mm., and appears to be zero above the latter pressure. If the foregoing discussion be sound, we may infer that the first layer of gas molecules on the surface is formed at a pressure lower than 0·06 mm., and that the film has attained its maximum thickness before a pressure of about 1·5 mm.

The upper curve in Graph 1 for air at atmospheric pressure shows that the bulk gradient is practically constant until within a distance of a few millimetres of the plates. We may conclude that the gas films for air under these conditions are not greater than a few millimetres.

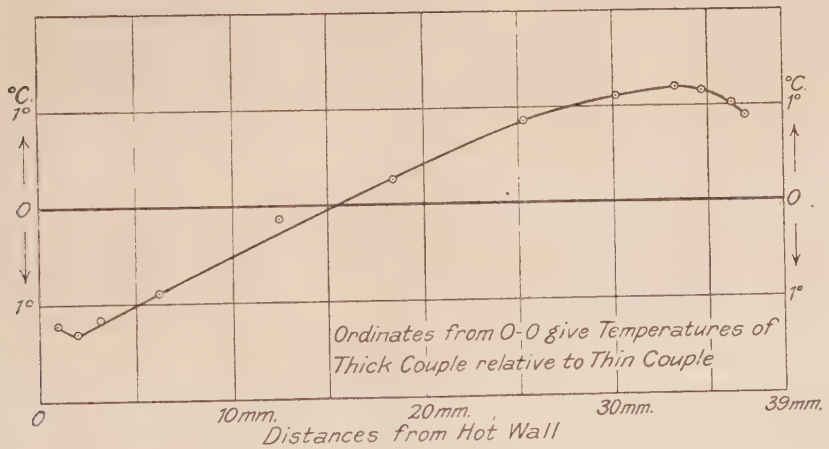
We may reasonably take the thickness of such a film as a qualitative indication of the degree of departure of the gas considered, from the perfection required by the ordinary kinetic theory, since the gas films get thicker as a result of the mutual attraction between the gas molecules. Thus the more nearly a gas approaches perfection the more nearly will the bulk gradient at high pressures approach the maximum gradient $(\theta_H - \theta_C) \div HC$. A comparison of the upper curves in Graphs 1 and 2 suggests that the mutual attractive forces amongst the hydrogen molecules

are much less than in the case of air—a fact well established from other considerations.

Appendix.

In the foregoing Paper it was pointed out that although only the results for the thin thermocouple are considered in the tables and graphs, results were also obtained for the thick couple in carrying out the experiments described. It was further pointed out that both thermocouples were calibrated together against a mercury-in-glass thermometer in a black-body enclosure over the whole range of temperature employed. It is clear that if there be no effects due to a difference in curvature of the surfaces there will be no difference in the indications of either couple.

In the actual experiments, however, a small but definite and systematic difference was found to exist between the two couples in air at atmospheric pressure. An example of the results is shown in Graph 6, where the difference in degrees



Graph 6.

centigrade between the two sets of readings for the two couples is plotted against different distances from the hot plate, when the hot and cold plates were maintained at temperatures 179°C. and 49°C. respectively.

It will be seen that the thick thermocouple indicates a lower temperature than the thin couple in the hotter portion of the chamber, but a higher temperature than the thin one in the colder portion. This interesting phenomenon was not observed in the case of air at low pressures, and was barely perceptible in the case of Hydrogen at any pressure.

In view of the important bearing this matter may have on the determination of temperatures in unequally heated enclosures at ordinary pressures, further experiments are being carried out, and it is hoped shortly to publish a second Paper on the subject.

It is highly probable that the explanation lies in the variation with curvature of the thickness of the gas film which exists at the surfaces of the wire thermocouples.

In conclusion, we wish to express our thanks to Prof. G. A. Schott, who originally drew our attention to the problem of the temperature discontinuity existing at the surface between a solid and a gas. We especially wish to thank Prof. G. Owen for his stimulating interest and advice throughout the course of the experiments.

DISCUSSION.

Mr. B. S. Gossling said that the temperature in a space containing gas at low pressure was one of those delicate and uncomfortable subjects which should be mentioned only in carefully chosen society; he felt, however, that with the present authors it was in perfectly safe hands. He thought that the authors had dismissed the influence of radiation rather briefly. Radiation might be an important factor at a temperature of 150°C. As regards the case illustrated in Fig. 3, where the absence of molecular collisions was assumed, an exactly similar temperature distribution would be obtained in a perfect vacuum, owing solely to radiation. Must not the authors' curve, therefore, be regarded as determined by radiation and not by conduction? There is one objection to such a view: it would imply that very near to the hot plate the thermocouple should register a temperature four times closer to the hot than to the cold temperature, assuming the fourth power law of radiation; whereas in graph 1, for instance, the experimental curve lies nearer to the cold than to the hot temperature. At the end of their theoretical discussion the authors speak of the Langmuir film. The word "film" which has come into use in this connexion seems a misnomer, as it denotes a region 4 mm. thick—viz., that in which convection ceases and conduction only is effective, a discontinuity at the edge of this region being indicated by optical experiments. The authors' discussion of a really thin film is very interesting, but such a film is quite different from the Langmuir layer. The continuation of the authors' work might throw some new light on the nature of the adsorption of gases. An interesting case would arise where the hot plate was incandescent, but the difficulties involved in an experimental investigation of it would be considerable.

Dr. Ezer Griffiths: I think it is a pity that the discussion of the methods of deducing true temperatures from the indications of the thermocouples should have been deferred, for it is difficult to assess the value of the theoretical edifice which has been built up unless one is satisfied as to the experimental foundation. I have not had the opportunity of studying the Paper in detail, but glancing through it I find no reference to two Papers by G. H. Henderson in the Physical Review for 1920, entitled: "A new method of determining the temperature variation of the thermal conductivity of gases," and "The determination of the radiation error in the measurement of gas temperatures." Henderson's method is similar to that of the authors, but his thermocouples were of much finer diameter, 1 mil as compared with the 5 mils of the present Paper. One of Henderson's remarks in his conclusions might be quoted: "One point is clearly brought out in the present work, viz., the unreliability of temperatures as indicated by a temperature-measuring device placed in a gas in the presence of surfaces at temperatures differing considerably from that of the body of the gas." I would suggest that the authors should make a careful intercomparison of their work with that of Henderson, for he seems to have devoted much time to the problem of the measurement of temperature at a point in the gas. Personally I should favour a method of determining the temperature gradient by some interference method utilising the change of refractive index of the gas with temperature. As regards Mr. Gossling's remarks concerning the Langmuir film theory, this has received considerable modification and I would refer him to a Paper by Chester W. Rice in Journ. Amer. Inst. of Elec. Eng., 1923.

Mr. Gossling agreed that the Langmuir theory had been modified, but pointed out that the boundary of the Langmuir region now assumed to divide the turbulent from the streamline motion in the gas, occurs at a definite distance from the solid surface. Interference methods of measuring temperature lead to difficulties at low pressures, where "temperature" ceases to have a useful physical meaning.

Mr. C. R. Darling referred to the discrepancy between the readings of the two thermocouples, and suggested the use of flat strips similar to those employed by Dr. Moll (*see* Proc. Phys. Soc., Vol. 35, p. 257). The Langmuir layer need not be an actual layer; all that it implies is that in the region in question thermal phenomena are the same as they would be if such a layer existed. He had determined the temperature distribution in badly conducting solids, such as magnesia and asbestos, and obtained curves resembling Graph 1, except that they had not been extended

towards the outer edge. The curves if extended by extrapolation would indicate a temperature about 15° higher than the actual temperature of the hot wall. In these experiments the specimen was cylindrical and the heat flowed parallel to the axis. The thermocouples were buried, the wires being normal to the axis.

The PRESIDENT suggested that in view of the discussion the authors should add to the Paper a note on their temperature measurements.

AUTHORS' reply (communicated): (In reply to Mr. Gossling). We consider that the question of radiation might with advantage have been given more attention. The temperature gradient due to radiation alone in an apparatus of finite size is probably most satisfactorily examined experimentally. In our case, for example, the radiation gradient to be expected according to Mr. Gossling is not supported by experimental results (*see* Graph 3).

Although the relative effect of radiation on the indications of the thermocouples will increase with decrease of pressure, the question as to whether the curves at our lower pressures are determined by radiation rather than by conduction will depend on the absolute magnitude of the gradient due to radiation alone. In our Paper, we were inclined to consider this gradient almost negligible, especially since the gradient for our lowest pressure, which is still comparatively high, was small.

(In reply to Mr. Darling). The use of flat strips instead of wires for thermocouples might be a useful variation in the conditions, and therefore help to determine the magnitude of the effect of radiation.

(In reply to Dr. Griffiths). The question reduces to that of the efficiency of a thermocouple for the measurement of temperatures under the conditions of the experiment.

We are grateful to him for referring us to the Papers by G. H. Henderson, the more so since the latter refers to some previous work by Kreisinger and Barkley. A critical comparison of the Papers as suggested would, to be satisfactory, require a further Paper. It might, however, be pointed out that whilst we agree with Henderson's general analysis of the problem which deals with atmospheric pressure only, the assumptions made by him in deducing the numerical values of the corrections due to the radiation effect to be applied to the reading of the thermocouple seem too big to warrant much reliance being placed on those values when obtained. Further, the assumption of no discontinuity at the wall as a check on his calculations is hardly permissible.

It will be clear from the results in the Appendix that we had realised that the temperatures indicated by the platform thermocouples at any point could not be the actual temperature of the gas there; and that the thinner the couple, the less pronounced would be the temperature drops at the plates at atmospheric pressure. In view, however, of the small differences found to exist between the two couples, we assumed that the temperatures indicated were probably not far removed from the true values, although at the same time we stated our intention to inquire more closely into the matter. In this connexion, it is unfortunate that we were not able to use our third and thinnest platform couple, which was of similar diameter to that used by Henderson, especially since a consideration of his work and that of Kreisinger and Barkley inclines us to the view that the departures from the true temperatures are probably greater than we at first anticipated. This will, of course, cause the bulk gradient at atmospheric pressure to be steeper than that given in Graph 1. However, even if the corrections deduced by Henderson for bigger temperature gradients be applied to our results, there will still be a discontinuity at the plates at atmospheric pressure.

Although not raised in the discussion, there still remains the question of the possible effect of the side walls. We feel that this matter deserves some further consideration. One of us is at present continuing the general problem, using an apparatus of type similar to that described in our Paper, but of modified dimensions. Several thermocouples of varying diameters are being employed, and it is proposed to deduce by a graphical method the temperature for an infinitely thin couple. The other proposes studying the problem by a method involving the use of the principles of resistance thermometry. In this case the apparatus will necessarily be of different design, the source of heat being a hot wire. It may be possible with this apparatus to investigate the problem when the wire is incandescent as suggested by Mr. Gossling.

IV.—THE DIELECTRIC PROPERTIES OF MICA.

An Investigation carried out for the British Electrical and Allied Industries Research Association.

By D. W. Dye, *B.Sc.*, and L. Hartshorn, *A.R.C.S., B.Sc., D.I.C.*, The National Physical Laboratory.

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Abstract.

Two methods used for determining the dielectric constant of thin sheets of mica are described and the errors discussed.

The results of measurements on over 170 sheets of mica are given. The samples tested included best clear mica, ruby, green and brown in colour, stained and spotted samples, and also samples with gaseous inclusions. There were also samples of amber mica and silver amber mica.

The results show that practically all samples of muscovite (potassium mica) have the same dielectric constant, the most probable value being 7·0. The presence of inclusions has no great effect on this value. The samples of phlogopite (magnesium mica)—i.e., the amber micas—have rather lower dielectric constants, about 6·0 for normal amber mica, and 5·0 for the silver amber variety.

Values of power factor are given for many of the samples. For the clear micas (muscovite) it may be as low as 0·0003, and possibly less. The effect of inclusions is to increase it considerably. The power factor of the amber micas is greater than that of the muscovite variety—about 0·02 for the clear amber and 0·05 for the silver amber variety.

The samples were all about 8 cm. square and 0·1 mm. thick, and the measurements were all made on the Carey Foster bridge at a frequency of 800 cycles per second.

I. Introductory.

WHEN the present investigation was first started over two years ago, it was generally recognised that our knowledge of the dielectric constant of mica was very unsatisfactory. The values given by different investigators and collected in the various tables of physical constants show great variability. Values as widely different as 4 and 8 will be found, and generally speaking there is no obvious reason for such variability. Many condenser makers seemed to think that such large variations of dielectric constant did actually occur, and the differences were not due to experimental errors. It must of course be recognised that "mica" is not a definite chemical compound—i.e., there are many kinds of mica differing in chemical composition, colour and general optical and mechanical properties. These are obtained from various localities, including India, East Africa, Brazil, etc.; but there seemed to be no reliable information as to whether the differences in dielectric constant were greater between different samples from the same source, or between samples of the same kind from different sources. In view of this state of affairs, the British Electrical and Allied Industries Research Association included in its comprehensive programme of research on dielectrics an investigation into the dielectric properties of as wide a range of mica samples as could be obtained. The present Paper is an account of this research, which was carried out for the Association at the National Physical Laboratory, and the authors are indebted to the Association for permission to publish the results.

II. Methods of Measurement.

Probably the most common way of measuring the dielectric constant of material in the form of thin sheets is to form a condenser by pressing two sheets of tin foil

into contact with the sample, one on either side of it, and then to measure its capacity. This method has been used by several observers,* but it is well known that the great difficulty attending its use is that of securing intimate contact between the material and the electrodes. On this account electrodes of mercury instead of tin foil have frequently been used ; but even in this case, unless special precautions are taken, air bubbles are apt to form on the surface of the dielectric, and so vitiate the results.† Another method consists of making a parallel plate air condenser of known dimensions, measuring its capacity, then inserting a sheet of the material into the gap between the plates of the condenser, and measuring the change of capacity produced. This method has been used by A. Campbell.‡ It of course entirely avoids the above-mentioned difficulty. In the present investigation both the air gap and the mercury electrode methods are used.

(A) *The Air Gap Method.*

(*a*) *The Apparatus.*—The samples of mica were supplied by Messrs. F. Wiggins and Sons. They were all about 8 cm. $\times$ 8 cm. square and 0·1 mm. thick. Hitherto the air gap method does not appear to have been carried to the refinement necessary for its use in tests on such samples. The development at the Laboratory of means

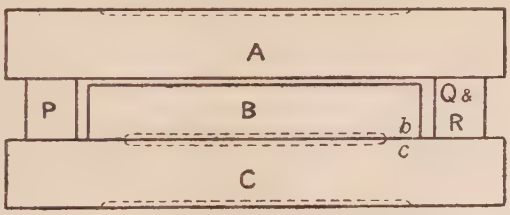

Fig. 1.

of producing surfaces which are flat and parallel to a very high degree of accuracy led us to attempt the construction of a standard air gap for these tests, the width of the gap to be from about 0·1 to 0·25 mm. In consultation with the Metrology Department, the form shown in Fig. 1 was arrived at.

The gap consists of three case-hardened and accurately-ground plane parallel steel plates. The upper plate *A* forms one of the electrodes, and its under surface is plane all over. The top plane surface is an annulus, and is necessary only for the production of the under surface. This plate *A* is supported by three clear natural quartz pillars, *P*, *Q* and *R*, made to an exact dimension, and each as nearly as possible of the same length. They have plane parallel faces. They were supplied ground to size and polished by Messrs. Adam Hilger. The base plate *C* has annular plane surfaces above and below. The other electrode consists of the smaller circular plate *B*, having a complete upper plane surface and an annular lower plane surface, the two being accurately parallel. The thickness of this plate determines the width of the air gap. By the use of plates *B* of various thicknesses a series of gaps of

* *See*, for example, A. Campbell, Proc. Roy. Soc., A, Vol. 78, p. 196 (1906). Fleming and Dyke, Journal I.E.E., Vol. 49, p. 323 (1912).

† *See*, for example, Bairsto, Proc. Phys. Soc., Vol. 25, p. 301 (1913). R. Appleyard, Proc. Phys. Soc., Vol. 19, p. 724 (1905). J. R. Weeks, Phys. Rev., April (1922).

‡ A. Campbell, Proc. Roy. Soc., A, Vol. 78, p. 196 (1906).

different widths is obtained. To secure sufficient stiffness of the plates to avoid the possibility of appreciable bending, they were made quite thick—viz., 2 cm. for plate *B* and 2·5 cm. for plates *A* and *C*. These two plates were each 15·5 cm. in diameter.

The lengths of the quartz pillars were measured by the Metrology Department and were found to be as follows :—

	Max. length at 62°F.
No. 1	$1{\cdot}9874_8$ cm.
2	$1{\cdot}9874_6$,,
3	$1{\cdot}9874_4$,,
Mean	$1{\cdot}9874_6$ cm.

The faces were all slightly concave, but the amount was quite negligible.

In the present series of measurements three different gaps were used. Their dimensions are given in Table I. The diameter of the gap is, of course, the diameter of the upper surface of plate *B*. In Gap No. 3 this was smaller than in the other two cases.

TABLE I.—*Constants of the Air Gaps.*

Mean height of quartz pillars $=1{\cdot}9874_6$ cm.

Gap No.	Thickness of plate *B*. cm.	Width of gap (*d*). mm.	Diameter of gap. cm.	Calculated capacity *C*. $\mu\mu F$.	Observed capacity C_1. $\mu\mu F$.	Edge capacity C_1-C.
1	$1{\cdot}9747_3$	$0{\cdot}127_3$	11·48	719	732	13
2	$1{\cdot}9625_1$	$0{\cdot}249_5$	11·48	367	382	15
3	$1{\cdot}9747_8$	$0{\cdot}126_8$	7·56	313	333	20

(*b*) *Manipulation of the Gap.*—In order that it may be possible to calculate the width of the gap from the thicknesses of the quartz pillars and the plate *B*, it is, of course, essential that there should be very good contact between the quartz pillars and plates *A* and *C*, and also between plates *B* and *C*. It was found to be very easy to get good contact between the quartz and steel faces, and observation of the interference fringes soon showed when this was attained. It was merely necessary to put the pillar on the steel surface and work it about until the coloured interference fringes disappeared. Good contact between the two steel surfaces *b* and *c* of plates *B* and *C* was not so easy to obtain. If the surfaces were quite dry, there was always an air film between them. To get good contact the surface *C* must be covered with a thin layer of paraffin oil. Plate *B* is then placed in position and worked about. The oil is gradually pushed out from between the contact faces and the contact becomes more and more perfect. When the paraffin layer is so thin as to appear black, the contact is sufficiently good. When all the contacts are good, there is a very considerable force acting across the contact faces and holding the components together, so that the whole apparatus may be lifted by the top plate *A*.

The capacity of the apparatus may be written

$$C_1=C_g+C_e$$
$$=\frac{A}{3{\cdot}6\pi d}+C_e \text{ micro-microfarads,}$$

where the first term represents the actual capacity across the air gap and, the second

C_e represents the "edge" capacity. The actual values for the three gaps used are given in Table I.

The edge capacity of No. 3 is larger than the others, because in this case the sides of plate B are sloping, and plate A overhangs the gap by a greater amount.

The accuracy of reproduction which could be obtained by setting up the apparatus on different days may be judged from the values given in Table II obtained for the capacity of Gap No. 2 on different occasions.

TABLE II.

Date.	Observed capacity of Gap No. 2.
1st February	382_4 $\mu\mu F$
2nd ,, ...	381_7
3rd ,, ...	381_5
4th ,, ...	381_7
14th ,, ...	381_5
30th March	381_4

The imperfection of contact between plates B and C always manifests itself by producing a larger capacity, owing to reduction in the length of the gap.

When not in use the highly polished surfaces were preserved by being kept under paraffin.

(*c*) *Method of Test.*—The gap having been set up as described above, its capacity (C_1) was measured. One of the mica samples was then placed in the air gap, and the capacity (C_2) again determined. From these two observations the dielectric constant of the mica sample is deduced. These capacities may each be considered as made up of two portions. Expressing the capacities in micro-microfarads, we have

$$C_1=\frac{A}{3\cdot6\pi d}+C_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where A = area of mica in the air gap in cm.^{2}.

d = width of air gap in cm.

C_3 represents that part of the air gap outside the area A occupied by the mica, and, of course, includes the "edge capacity." Also neglecting edge effects at the boundary of the mica

$$C_2=\frac{A}{3\cdot6\pi\left[d-\frac{t(K-1)}{K}\right]}+C_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where t = the thickness of the mica sample,

and K = dielectric constant of the sample.

Eliminating C_3 from (1) and (2), we obtain

$$K=\frac{C_0+(C_2-C_1)}{C_0-\frac{d-t}{t}(C_2-C_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $$C_0=\frac{A}{3\cdot6\pi d}$$

C_0 represents the capacity of the area A of the gap, which the mica would occupy, before the mica is inserted.

In the case of Gaps Nos. 1 and 2, the samples of mica lay entirely within the gap, and thus A was the actual area of the mica sample. Gap No. 3 was designed so that when the mica was in position, it projected out of the gap on all sides. In this case A is the area of the upper surface of the middle plate. This third gap was made with a view to the determination of the power factors of the samples, as it was found that when the mica only partly filled the gap, dust particles bridged across the gap outside the mica and vitiated the results. By allowing the mica to occupy the whole area of the gap, this trouble was removed and the leakage avoided.

In order to obtain an idea as to the accuracy obtainable by this method, three samples of best clear ruby mica were tested in each of the three gaps. The results are given in Table III.

TABLE III.

Sample No.	Dielectric Constant.			Mean.
	Gap No. 1.	Gap No. 2.	Gap No. 3.	
1	6·4	7·0	6·6	6·7
2	6·3	6·6	6·5	6·5
3	6·3	6·8	6·5	6·5

(*d*) *Sources of Error.*—It is to be noted that four quantities are required in order to determine K, viz., A (C_2-C_1), d and t, and errors in the determination of these affect the value of K by varying amounts. By differentiation of (3) with respect to each of these quantities we can find the percentage error in K resulting from a 1 per cent. error in A, C_2-C_1, d and t. The results are found to depend on the thickness of the specimen tested. In the case of a typical sample 0·094 mm. thick, the results are as follows (Table IV) :—

TABLE IV.

Gap No.	Percentage error in K due to a + 1 per cent. error in			
	A	C_2-C_1	t	d
1	—2	+2	—5·5	+7·5
2	—4	+4	—5·5	+9·5
3	—2	+2	—5·5	+7·5

Thus it is a disadvantage of the method that errors in the determination of d and t cause comparatively large errors in the value of K. In order to determine t as accurately as possible, a special optical lever arrangement described below was used.

(*e*) *Determination of Thickness.*—It was at first anticipated that the thickness of the mica sheets would be uniform to a considerable degree of accuracy, in the case of the clear mica at least, but on examination of the samples between the flat faces of two quartz pillars, the interference fringes showed that there were local variations of thickness over a circle as small as 1 cm. in diameter. Thus it was necessary to use a measuring device which would give the thickness at a point on the sheet, or at least the average over a very small area, and not merely the maximum thickness over an area as large as 1 cm. A measuring machine having ball contact faces was tried, but the pressure was found to be too great, and an optical lever was

designed in which the pressure was quite small. It is shown in Fig. 2. The lever is a bar of steel $13 \times 1{\cdot}8 \times 0{\cdot}5$ cm., having polished steel balls AA, 6 mm. diameter, let into it at each end, and a similar ball B in the middle, the centre of B being displaced about 1 mm. from the line of centres of AA. Three gauges, a, a, b, of equal height, are wrung down on to a flat steel plate, and arranged so that the balls AAB can rest on their upper surfaces as in Fig. 2. The lever can then turn about the line joining the points of contact of AA, and does so until the ball B comes into contact with the surface of the gauge underneath it. If, then, a mica sample is placed on the middle supporting gauge, and the lever allowed to come into contact, it is tilted from its zero position, and the angle of tilt is a measure of the thickness of the sample. The angle was measured by the usual lamp and scale arrangement, and the scale was calibrated by using standard gauges of known value for the middle support. On the scale a deflection of 40 cms. corresponded to a thickness of about 0·1 mm. The readings were reliable to less than 0·5 mm. on the scale, so that readings could be made to 0·0001 mm. The middle supporting gauge could not be spherical owing to the displacement of the point of contact when the lever tilts by the rolling of the balls AA. The most satisfactory form for this support was found to be a gauge ending in a pin 1 mm. diameter having a flat top (*see* Fig. 2). The lever was provided with an arrestment somewhat resembling that of an ordinary

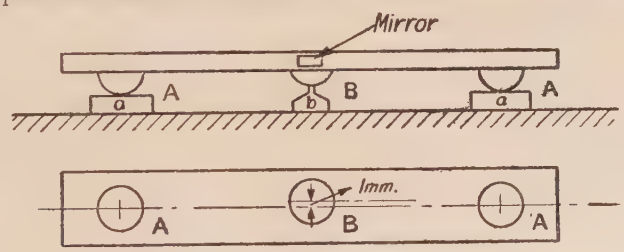

FIG. 2.

chemical balance. This allowed it to be raised for the placing of the mica in position and then lowered into the same position on the supporting gauges. The scale being made direct reading, it was possible to take a great number of observations very quickly. Nine measurements were made on each sheet at points uniformly distributed over its surface, and the mean taken. A few samples were found to be of practically uniform thickness, but generally there were variations. Sometimes a definite line of discontinuity of thickness was found on a sample of clear mica, the portions on either side of the line being fairly uniform, but of different thicknesses. In other cases the thickness was very variable all over the sheet (this was especially noticeable in the case of mica with inclusions), so that it was impossible to get a value with any accuracy. A 10 per cent. variation of thickness would occasionally be found on one sheet. These results clearly indicate the danger of the formation of air pockets when condensers are built up with tin foil electrodes.

(*f*) *The Probable Errors.*—The value of the mean thickness was probably determined to an accuracy of 0·5 per cent. for the better samples. This corresponds to an error in K of rather less than 3 per cent. for all the gaps.

The value of d was obtained from measurements of the thickness of the quartz pillars and the middle plate. An idea of the accuracy obtained is given by the

reproducibility of the gaps. It was probably about 0·25 per cent. for gaps 1 and 3, and 0·12 per cent. for gap 2. This corresponds to errors in K of 2 per cent. for gaps 1 and 3 and 1 per cent. for gap 2.

The errors in A and (C_2-C_1) are of comparatively small importance in gaps 1 and 3, and together will probably cause an error of less than 1 per cent. in K. In the case of gap 2, C_2-C_1 is considerably smaller, and an error of 0·5 per cent. in its measurement is not unlikely. This will cause an error of 2 per cent. in K.

Thus the maximum estimated errors which could occur in the value of K may total 6 per cent. for gaps 1 and 3, and 7 per cent. for gap 2. Typical results on three samples have been given in Table III. The maximum deviation from the mean value is seen to amount to 6 per cent. in the case of gap 2 and 4·5 per cent. in the case of gaps 1 and 3.

(B) *The Mercury Electrode Method.*

(*a*) *Apparatus.*—As previously mentioned, the chief difficulty encountered in using mercury electrodes is the formation of air bubbles on the mica surface. Several forms of electrode were tried with a view to eliminating this trouble. They are shown in Fig. 3.

The first form (*a*) consists of two recessed ebonite plates bolted together and provided with rubber rings, B, to form a mercury tight joint with the mica, which was clamped between the plates. The mercury was poured in through holes in the top of the ebonite plates. On shaking up this arrangement it was noticed that the capacity sometimes increased in value. This indicated that air bubbles were formed. In order to study the mercury-mica contact, one of the ebonite plates was removed, and a mica sheet was pressed up against the rubber ring of the other plate by means of a sheet of glass. On pouring in the mercury, the formation of air bubbles could be seen, and it was found to be due mainly to splashing as the mercury entered. Of the various arrangements tried in order to avoid this, the most successful is shown at Fig. 3 (*b*). The mercury is admitted through fine bore glass tubing entering the ebonite plate from below. A hole in the top of the ebonite plate allows the air to escape. The best size of glass tubes was found by trial (about 1 mm. bore). When this was used, the mercury was seen to enter slowly, and to gradually flow up the mica surface, forming no bubbles as long as the surface was free from dust. On fitting the same sample a number of times into this arrangement, the capacity readings could be repeated with a maximum discrepancy of about 1 per cent., which might easily be due to the yielding of the rubber rings.

A pair of amalgamated copper blocks, Fig. 3 (*c*), was also used. These were 4·5 cms. in diameter and 1·25 cms. thick. Their surfaces were carefully made flat. They were then amalgamated and flooded with mercury. By sliding the mica on to the lower copper block A very carefully it was found possible to get rid of bubbles (when any did form, they could of course be seen through the mica). The top block B presented more difficulty. It was pressed down on to the mica surface, and the excess of mercury removed with a camel hair brush. The blocks so assembled clung together with considerable force, and it was possible to invert the arrangement, remove the block A, and examine the contact surface of the block B. Usually there were air bubbles, and it is unlikely that results could be obtained reliable to less than about 3 per cent.

Experiments were also made with the upper block hollow, and with a narrow

rim 1 mm. wide, resting on the mica. The inside of the block was filled up with mercury. (*See* Fig. 3 (*d*)). Here, again, the air bubble difficulty could not be entirely overcome. The mica samples were all tested in the ebonite clamps, Fig. 3 (*b*), which is considered the most satisfactory arrangement.

(*b*) *Errors.*—If C represents the capacity in micro-microfarads of the condenser formed by the mica sample, then, neglecting the edge capacity effect, we have for the dielectric constant

$$K=\frac{3{\cdot}6\pi Ct}{A}$$

where t is the thickness of the sample, and A is the area of either electrode. We

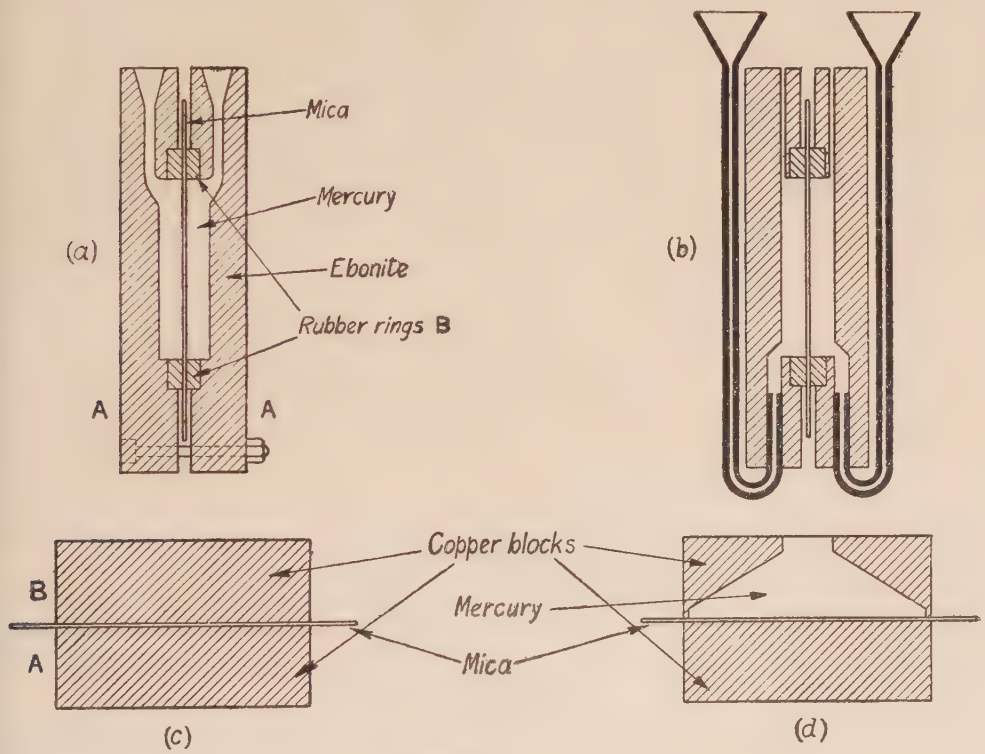

FIG. 3.

notice immediately that this method possesses the advantages (1) that the calculation is much simpler, (2) that errors in C, t and A cause percentage errors in K of only the same amount, whereas by the air gap method all these errors were magnified. The advantages possessed by the air-gap method are (1) there is no difficulty about air bubbles, (2) the edge capacity effect is almost entirely eliminated.

The diameter of the mercury electrodes used (Fig. 3 (*b*)) was 5·19 cm. and their thickness 5 mm. The capacity of the condenser formed when a mica sample was inserted was about 1,400 $\mu\mu F$. An estimation of the edge capacity effect was made, using Kirchoff's formula, but it was considered to be not more than $5\mu\mu F$,

which is negligibly small for these experiments. The area A was slightly indefinite owing to the yielding of the rubber rings under the clamping pressure, but the error was probably not more than about 1 per cent. Thus the total error to be expected in K found by this method is about 2 per cent., as against 5 or 6 per cent. by the air-gap method.

It must be emphasized that the chief feature of this investigation is the great number of samples examined. More accurate results could easily be obtained on any one kind of mica by carefully choosing a piece of uniform thickness and of a size that could be accurately measured. The uncertainty in the area A of the mercury electrode owing to the yielding of the rubber can be avoided in the case of such a sample by doing away with the rubber and holding the mica sheet between

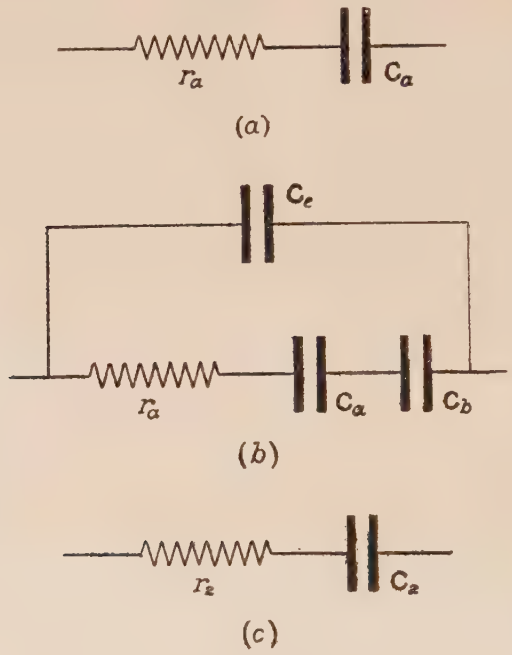

FIG. 4.

two narrow unyielding rims; but when the thickness of the sample is not uniform the rubber rings are necessary to make the arrangement mercury tight. Over 170 samples were tested by us, and, as previously mentioned, many of them were not of uniform thickness. In testing such a large number of samples the air-gap method has the advantage that, having set up the gap, the mica sheets can be slipped in one after the other and the necessary observations made very quickly. This is one compensation for the somewhat large errors which the method involves. In view of the fact that various observers have obtained such widely different values for the dielectric constant of mica, considerable importance is attached to the fact that most of these samples were tested by both methods, and that the agreement obtained is not entirely unsatisfactory.

(C) *Effect of Conduction and Power Loss.*

Equation (3), giving K in terms of the readings obtained with the air gap, is obtained on the assumption that the dielectric sample is flat, of unform thickness and composition, and everywhere at right angles to the lines of force crossing the gap—i.e., parallel to the sides of the gap. No sample will strictly fulfil these requirements, but in the case of micas of good quality this is not likely to cause serious errors, provided that the true mean thickness of the sample is used. The general consistency of the results on the clear micas is sufficient indication of this.

It is further to be noticed that the derivation of (3) assumes that the potential gradient in the dielectric sample depends only on its dielectric constant—i.e., no account is taken of conduction or absorption in the dielectric. The effect of both these properties is to cause a dissipation of power in the sample under the applied alternating voltage. All the tests were made on the Carey Foster bridge at a frequency of 800 cycles per second. Thus what is actually measured in any one case is the combination of perfect condenser C_a and series resistance r_a (Fig. 4), which is equivalent to the test condenser, under the conditions of test. If r_a, C_a is the equivalent of the mica sheet alone, then, when it is inserted into the gap, the arrangement is equivalent to Fig. 4 (*b*). Here C_b represents the effect of the air space between the mica sheet and the steel plates, and C_e represents the edge capacity. The bridge actually measures the effective resistance r_2 and capacity C_2 of this arrangement (Fig. 4 (*c*)). It is easy to show that for the equivalent arrangements Fig. 4 (*b*) and (*c*) we must have

$$r_2 = \frac{r_a \cdot \dfrac{1}{\omega^2 C_e^2}}{r_a^2 + \dfrac{1}{\omega^2}\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}+\dfrac{1}{C_e}\right)^2} \quad \ldots\ldots\ldots \quad (4)$$

and

$$\frac{1}{C_2} = \frac{\dfrac{1}{C_e}\left[r_a^2 + \dfrac{1}{\omega^2}\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}\right)\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}+\dfrac{1}{C_e}\right)\right]}{r_a^2 + \dfrac{1}{\omega^2}\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}+\dfrac{1}{C_e}\right)^2} \quad \ldots\ldots \quad (5)$$

In practice r_a^2 is very small compared with

$$\frac{1}{\omega^2}\left(\frac{1}{C_a}+\frac{1}{C_b}\right)\left(\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_e}\right)$$

and thus we have approximately

$$r_2 = r_a \cdot \frac{\left(\dfrac{1}{C_e}\right)^2}{\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}+\dfrac{1}{C_e}\right)^2} \quad \ldots\ldots\ldots \quad (6)$$

$$\frac{1}{C_2} = \frac{\dfrac{1}{C_e}\left(\dfrac{1}{C_a}+\dfrac{1}{C_b}\right)}{\dfrac{1}{C_a}+\dfrac{1}{C_b}+\dfrac{1}{C_e}} \quad \ldots\ldots\ldots \quad (7)$$

giving

$$r_a = r_2\left(\frac{C_2}{C_2 - C_e}\right)^2 \text{approx.} \quad \ldots\ldots\ldots \quad (8)$$

Equation (3) can be deduced from (7), and thus holds to the same degree of approximation. By the mercury electrode method C_a and r_a are determined directly. By the air-gap method r_a is obtained from (8), and C_a from (7) or (3), which is equivalent to (7).

(D) *Power Factor Determinations.*

At the same time as the capacity measurements were made the values of the effective resistance r_a were also determined, and hence the power factor of the sample deduced in the usual manner from

$$\text{Power factor} = r_a C_a \omega \text{ (approx.)}.$$

In the case of the clear micas the power factor was found to be very small, and the reading obtained was very dependent on the condition of the experiment—e.g., when mercury electrodes were used—a higher value of power factor was obtained if the mercury became contaminated with impurities. Different results were also obtained for the same samples on different days, the changes being apparently due to changes in the humidity of the atmosphere. Values as low as 0·0003 were obtained for clear ruby mica, but owing to the difficulty of making sure that the power loss measured was actually dissipated in the mica, the power factor tests on the clear mica were deferred, and it has not been possible to take up the matter again yet. Power factor results for the mica with inclusions were obtained, as owing to the greater power loss in the case of these samples, the above-mentioned difficulties do not arise, and one of our immediate objects was to find out the effect of inclusions on the power factor.

III. Results.

All the samples tested can be divided into two mineralogical groups. They were either

(1) Muscovite (potassium mica) or (2) phlogopite (magnesium mica).

Inside these groups the specimens were divided into various grades. Of these it is convenient to consider two main divisions—

(*a*) Clear micas—i.e., samples which are quite transparent and free from all inclusions, gaseous or solid.

(*b*) Samples with inclusions.

Considering now the clear micas of the muscovite group. They were classed according to colour as ruby, green and brown micas, and, where possible, samples of each kind from various sources were tested. A sample of any one kind usually consisted of eight sheets, and separate tests were made on each sheet. It is unnecessary to give all the results in detail. The results given in Table V for Bengal ruby mica are typical.

It is to be noted that the result by each method is the same for every sheet within the experimental error of the method used, but the air-gap results are all somewhat lower than those obtained with mercury electrodes. All the samples of the clear micas of the muscovite group gave results very similar to those in Table V. Samples of different colour and from different countries all appear to have practically the same dielectric constant. The values given in Table VI represent in each case the mean for 8 sheets.

TABLE V.—*Dielectric Constant of Bengal Ruby Mica.*

Sheet No.	Dielectric Constant.	
	By Air Gap Method.	With Mercury Electrodes.
1	6·7	7·1
2	6·3	7·1
3	6·5	7·0
4	6·4	7·1
5	6·4	7·0
6	6·6	7·0
7	6·6	7·0
8	6·4	7·1
Mean	6·5	$7·0_5$

TABLE VI.—*Dielectric Constant of Clear Mica.*

Mineralogical Group.	Colour.	Source.	Dielectric Constant.	
			Air Gap.	Mercury Electrodes.
Muscovite	Ruby ...	Bengal	6·5	7·0
		Brazil	6·4	7·0
		Madagascar ...	6·4	7·0
		E. Africa	6·5	7·0
	Green ...	Madras	6·6	7·0
		E. Africa	6·7	7·0
	Brown ...	E. Africa	6·5	7·0

In the case of the samples with inclusions, values of both dielectric constant and power factor are given. Where possible, measurements were made both by the mercury electrode and the air-gap methods, and the results of both are given; but in some cases it was impossible to apply the air-gap method. The reason for this is that not only are the samples with inclusions far from uniform in composition, but they are also by no means flat, and when the mica is in the air gap the laminæ of its structure are not equipotential surfaces. Thus conduction occurs along the conducting streaks and patches constituting the inclusions (spots and stains), and the effective capacity of the whole arrangement becomes much larger than would correspond to formula (3). In fact, in many cases C_2 was so large that $(C_2-C_1)(d-t)/t$ was greater than C_0, and the value of K given by (3) was negative. That these effects are due to corrugations in the sample was proved by the fact that samples which gave impossible results in the wide gap gave reasonable results when placed in a narrow gap, which had the effect of flattening out the sheet. On account of these difficulties it would not be safe to trust the air-gap values for the spotted micas without the corroboration of the mercury electrode results. More weight is to be attached to these results in any case, owing to the smaller probable errors.

The results are given in Table VII. As in the case of the clear micas, about eight sheets of each kind were tested, and in the table the mean value is given. In addition, the maximum and minimum values of K and power factor of the sheets of any one kind are given. In the case of a sample of spotted mica, the power factor, and to a less extent the capacity, depends on the part of the sheet tested.

The inclusion of one particular spot in the test may make a large difference. On this account some difference may be expected between the air-gap and mercury electrode results, since the mercury electrodes allowed only the central portion of each sheet to be tested, whereas the sheet being placed in Gap No. 2 an average value for the whole sheet is obtained.

TABLE VII.

Kind of Mica.	Air Gap Results.				Mercury Electrode Results.			
	Dielectric Constant.		Power Factor.		Dielectric Constant.		Power Factor.	
	Mean.	Extreme Values.	Mean.	Extreme Values.	Mean.	Extreme Values.	Mean.	Extreme Values.
Muscovite— Bengal Ruby Clear B (with gaseous inclusions)	6·2	6·0 6·4	0·004	0·001 0·022	7·1	7·1 7·2	0·009	0·005 0·011
Ditto, slightly stained	6·5	6·2 6·8	0·003	0·001 0·005	7·1	7·0 7·2	0·012	0·004 0·036
Ditto, stained	6·4	6·1 6·8	0·016	0·002 0·064	7·1	7·0 7·1	0·006	0·009 0·003
Ditto, much stained	6·5	6·3 6·9	0·026	0·004 0·137	7·0	6·9 7·2	0·005	0·004 0·007
Ditto, spotted	No measurements possible				7·1	6·9 7·3	0·010	0·005 0·017
Madras green, stained	6·7	6·3 6·9	0·054	0·012 0·080	7·1	6·9 7·3	0·010	0·009 0·012
Ditto, spotted	6·7	6·4 7·1	0·002	0·001 0·004	7·0	6·9 7·0	0·001	0·001 0·002
East African, brown spotted	No measurements possible				8·0	6·8 9·8	0·025	0·003 0·070
Madras, brown spotted	6·4	6·0 6·8	0·026	0·001 0·133	7·1	7·0 7·3	0·004	0·002 0·008
Phlogopite— Canadian amber	6·5	5·5 7·5	0·060	0·04 0·11	5·9	5·7 6·1	0·022	0·012 0·052
Canadian silver amber	Measurements worthless				4·8	4·4 5·0	0·055	0·049 0·059

Considering the samples of muscovite, we see that the inclusions have comparatively little effect on the dielectric constant. The value obtained by the air-gap method is approximately 6·5, and that obtained with mercury electrodes is

7·0 or 7·1 for nearly all the samples tested, which include ruby, green and brown micas, stained, spotted, and with gaseous inclusions. These gaseous inclusions were not simply sheets of clear mica in which the laminæ had become detached in places due to mechanical treatment; the inclusions consisted of small circular patches resembling bubbles, and there were usually a large number of them on each sheet. Only in the case of the sheets of East African brown spotted mica was there any considerable variation of K. In this case, values of 6·8 to 9·8 were obtained with mercury electrodes; but even here the samples of low power factor had a dielectric constant of about 7·0. The samples with high values of K were those of high power factor.

The samples of phlogopite—the amber micas—stand out from the others. Sheets of two kinds were tested:—

(*a*) Normal phlogopite, or ordinary amber mica; and
(*b*) The calcined variety, or silver amber mica.

The power losses in both these kinds are considerably greater than in the case of the muscovites, especially in the case of the silver amber samples.

The dielectric constant is lower than that for the muscovite samples, about 5·0 for silver amber mica, and 6·0 for ordinary amber mica, as against 7·0 for all samples of muscovite.

The differences in the values of the dielectric constant given by the two methods have not yet been satisfactorily explained. We have no reason to doubt that the effective values under the conditions imposed by the two methods are as given in the tables, and the differences are greater than the probable errors would lead us to expect. It is possible that there may be a contact effect at the mercury-mica surface. On the other hand, there may be a constant error in the air gap owing to concavity of the surfaces or to the small gaps at the contact of the quartz pillars and the plates. A few tests were made on samples of varnished cloth in the same apparatus. In this case the differences in the values of K obtained were far larger than for mica. Owing to the pressure of other work, we have not been able to continue this investigation, but we hope to resume it at some future date. It may be that there is an electrode capacity effect somewhat similar to that occurring in electrolytes.

In conclusion, we wish to acknowledge our indebtedness to the firm of Messrs. F. Wiggins & Sons, who supplied the mica samples, together with information as to the kinds of mica, and the localities from which they were obtained.

DISCUSSION.

Mr. B. P. DUDDING asked for fuller information as to the forms, colours and other physical properties of the various types of mica examined.

Mr. P. R. COURSEY, referring to Table VII. of the Paper, pointed out that the variation in the S.I.C. of different samples of mica of the same type was as great as the variation between samples of different types, and said these variations are perhaps to be accounted for by the fact that the thickness as measured may include the thicknesses of layers of included substances in addition to the mica. This view is confirmed by comparing an optical measurement of the thickness (by means of polarised light) with a mechanical measurement, when a considerable discrepancy is found. A Paper published some years ago in the Physical Review gave a large number of results rather greater than 8·0 for K; possibly the lower values found by the authors may be attributable to inclusions. It is known, for instance, that there are oil inclusions in the mica commonly employed for commutator rings, and, of course, in such cases it is the actual S.I.C. of the impure mica that is of practical importance. The power factor must be attributed

to laminæ of imperfectly insulating impurities, and in commercial samples the power factor becomes serious at radio-frequencies, when the apparent S.I.C. may drop to about 3.

Mr. F. E. Smith said that the air-gap method appealed to him as a delicate one, but the authors appeared to prefer the mercury-electrode method. He would suggest that the results of the latter might be vitiated by the fact that the actual area of contact of the mercury with the mica is not the same as the area of the exposed portion of the mica, in consequence of the meniscus effect. The cleaner the mercury the more marked would this discrepancy be, and the error in diameter arising from this cause might well amount to 1 mm., which would account for the 6 per cent. difference between the results of the two methods. He would suggest trying the effect of compressing the mercury.

Dr. D. Owen said that the Paper represented an accurate and interesting piece of work. Such limitations of accuracy as did occur in the first method were, admittedly, imposed by the extremely small thickness of the specimens ; and this notwithstanding the special care taken in planing and paralleling the metal plates bounding the air-gap. Was there some special virtue in the low value, 0·1 mm., which had been adopted ? Further, in regard to the question of uniformity of the specimens, it would be of interest to know whether the particles of possible foreign inclusions were large enough to be put in evidence by optical tests.

Mr. R. S Whipple suggested that some uncertainty arose from the fact that the mica was subjected to the hydrostatic pressure of the mercury, the effect of which would depend on the quantity of air inclusions.

Dr. A. B. Wood suggested that a further element of uncertainty was introduced by the fact that the rubber packing would bulge under pressure to an unknown extent, and so affect the exposed area of the mica.

Mr. G. L. Addenbrooke said that in his early work he had had some trouble with air bubbles, but had found that, under a pressure of 2 in., mercury will fill up any corner. He personally had used an average pressure of 4 in. or 5 in., and given the container a few sharp knocks to shake the mercury into the corners. Was the mica used by the authors dry ? Any dampness would have a considerable effect on the power factor. Under a microscope mica is distinctly seen to be laminated, and it may contain air films. All such points were, however, of subsidiary interest ; it was particularly interesting to the speaker to see the uniformity of the results obtained by the authors for a great variety of specimens, because he had always suspected that the lack of uniformity in the behaviour of mica condensers was to be attributed to other causes than variation of the S.I.C. ; he had been kept back from pressing this view publicly by the great variation to be found in the values given in tables of physical constants, a variation which was now shown to be erroneous. It was worth mentioning that Post Office engineers assume an all-round value of 6·6 for the purpose of their specifications. He hoped that this valuable Paper, besides clearing up the doubt as to the constancy of the S.I.C. of mica, would call attention to the general necessity for going over the accepted values of such constants again. Existing tables take no account of the power factor.

Mr. A. Campbell (communicated) : The Paper removes most of the uncertainty which existed regarding the dielectric constant of mica, and the authors are to be congratulated on the skill and patience with which they have carried out a difficult investigation. One of the main difficulties lies in the very small thickness of the material which has to be tested ; to obtain moderately accurate results the aid of the most refined metrology has to be called in. The writer used the air-gap method in 1906, but the formula he employed was incorrect (due to an algebraical slip). The error was not noticed at the time, as with the particular dimensions used it had very little effect on the results. The correct and complete formula is given by the authors. It is difficult to account for the systematic differences shown by the two methods. They are possibly due to the effect of surface moisture enlarging the effective area in the mercury electrode method. This might be investigated by using solid or liquid paraffin at the joints or by working in a desiccator. The results given for dielectric losses clearly indicate that in making high-class condensers the very best mica must be selected.

Authors' reply (communicated) : We do not think that the discrepancy between the two methods is due to uncertainty in the area of the mica-mercury contact surface, either on account of the meniscus effect or the yielding of the rubber rings. As mentioned in the Paper, the contact surface was examined by replacing one ebonite clamp by a sheet of plate glass, which was pressed up against the mica. In this way the meniscus effect and yielding of the rubber could actually be seen, and their effect on the result estimated. Further, in several cases, the area of the

mercury contact was measured by observing the contamination mark left by the mercury on the mica surface. We have no reason to doubt that the value of the area taken was correct within the limits specified. The possibility of surface moisture enlarging the effective area of the electrodes was also considered by us, mainly in connexion with the power-factor measurements. One set of such measurements was made in an enclosure containing dry air, and in another case, using the amalgamated copper blocks, melted paraffin was run over the exposed part of the mica surface. The effect on the apparent power factor was considerable, but on the value of K it was negligible compared with the general experimental errors. We did not include an account of these experiments in our Paper as we were not satisfied with our measurements on the power factor of the best clear mica, and hoped to deal fully with the matter at some future time.

In order to investigate the part played by moisture, some of the samples were tested and then placed (*a*) in dry air for five days, or (*b*) in moisture-saturated air for five days, or (*c*) heated to 35°C. for five days, and then retested. In each case the value of K was the same after this treatment, within the experimental error.

We were aware of the fact that Weeks had obtained values for K which are rather higher than ours, but after a careful consideration of the matter we could not account for this by any inaccuracy in our measurements. We certainly do not think our lower results are due to inclusions. The excellent optical properties of many of our samples were a sufficient guarantee against this. Our value of 7·0 is supported by those of Mattenklodt,* who, it is interesting to note, made his experiments with silver electrodes, i.e., his sheets of mica were silvered. His four tests gave $K=7{\cdot}1_8$, $7{\cdot}0_6$, $7{\cdot}0_9$, $7{\cdot}6_9$.

The thickness of 0·1 mm. was not definitely chosen by us. Sheets of this thickness happened to be available for the investigation and, in view of the fact that a thickness of this order is used in condensers, it was considered that results on such sheets would be of most value. It must be remembered also that the inclusion of air films is more likely to occur in thick specimens than in thin ones. Any further information required on the colours, forms, &c., of the various types of mica could probably be obtained through the British Electrical and Allied Industries Research Association.

Recent optical tests made for us in the Optical Department of the National Physical Laboratory have shown that the top steel plate is slightly hollow in the central region ; we are of the opinion that this defect, though small, is the cause of the systematic difference between the air-gap method and the mercury-electrode method. The plate will be rectified and a few further measurements should confirm this opinion.

With regard to the effect of the hydrostatic pressure of the mercury on the air inclusions : The mercury was only just above the highest point of the circular area of the electrode and so represented an average pressure of only 4 cms. of mercury. Further, owing to the thinness of the sheets and the good optical appearance of them, it is not thought that air inclusions of any consequence are present.

* Annalen der Physik, Vol. 27, p. 359 (1908).

V.—THE X-RAY EMISSION OF ELECTRONS FROM METAL FILMS, WITH SPECIAL REFERENCE TO THE REGION OF THE ABSORPTION LIMIT.

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Abstract.

From both theoretical and experimental viewpoints it is established that the kinetic energy of emission of electrons from a substance exposed to X-rays may be

$$(\tfrac{1}{2}mv^2)_{K,\,L,\,M,\,\text{etc.}} = h\nu - h\nu_{K,\,L,\,M,\,\text{etc.}}$$

where $h\nu$ is the energy of the incident quantum and $h\nu_{K,\,L,\,M,\,\text{etc.}}$ is the energy associated with one of the "quantum absorption limits" of the substance. As the K quantum limit has greater energy than the L.M., etc., limits, it follows that the K β-rays should have smallest energy. Hence arises the difficulty in giving a quantitative explanation of the selective increase in ionization by collision and absorption when ν exceeds ν_K.

The problem has been approached experimentally by Barkla and Miss Dallas, who have shown how the total energy of the β-rays from silver (as measured by the ionization produced in H_2) varies with wave-length as the region of the K absorption limit is crossed. The author has now determined directly the aggregate number of β-rays of all types emitted over the same range, and shows thereby, using his own previous observations, and also those of the authors quoted, that no selective change in the average energy occurs when the K fluorescent spectrum is excited. It is suggested that this is brought about by a re-grouping amongst the numbers and energies of the β-ray constituents, together with the addition of the K group of minimum individual energy as is required by the above equation.

The experimental work has necessitated the separation of the β-rays from the secondary effects they produce, the chief of which is the production of δ-rays of very low energy. The method of this separation is dealt with and the origin of the δ-rays discussed from the points of view of (*a*) thermionics, (*b*) recoil electrons, recently brought into prominence by Compton and C. T. R. Wilson, (*c*) impact electrons. δ-rays are impact electrons whose most frequent energy is little greater than 3·5 volts, which seems to be independent of the wave-length producing them, and of the substance from which they come. There are indications that they are influenced, at the moment of their departure from the surface, by the scattered X-rays.

The Problem Presented by Selective Absorption and Ionisation.

An attempt has been made to investigate by a new experimental method one of the most intricate problems associated with the absorption of X-rays and the re-emission of the energy in the form of electrons. It is now definitely established that there exists in the mixed stream of electrons emerging from a metal irradiated with X-rays of frequency ν groups whose energies are

$$\tfrac{1}{2}mv^2 = h\nu - h\nu_{K,\,L,\,M,\,\text{etc}} \quad \cdots \quad (1)$$

where h is Planck's constant and $h\nu_{K,\,L,\,M,\,\text{etc.}}$ is the energy associated with one of the quantum limits of the series of emission spectra of the absorbing substance.*

This equation implies that the energy of emission of these photo-electrons may have discrete values ranging from zero, if $h\nu_{K,\,L,\,M,\,\text{etc.}}$ happens to be exactly equal to $h\nu$, up to a maximum of $h\nu$ if the energy of binding of the ejected electrons to the atom from which it is ejected is negligible.

* For references *see* Simons, Phil. Mag., Vol. 46, p. 473 (1923). Also H. Robinson and F. W. Rawlinson, Phil. Mag., Vol. 28, p. 277 (1914); H. Robinson, Proc. Roy. Soc., A, 104, p. 455 (1923); C. T. R. Wilson, Proc. Roy. Soc., A, 104, pp. 1, 192 (1923).

For a given substance $h\nu_{K, L, M, \text{etc.}}$ may have several different values. It has been established that there are one "*K*," three "*L*," five "*M*," seven "*N*," five "*O*" quantum limits in descending order of energy* and corresponding to these there may be "*K*," "*L*," "*M*," etc., groups of photo-electrons in ascending order of energies. In particular the experiments of Sadler,† Beatty,‡ Barkla and Shearer§ and the author‖ showed that there were always present some electrons possessing practically the whole quantum of incident energy whether or not the *K* spectrum were excited in the absorbing substance; but neither these experiments nor those of De Broglie¶ and Whiddington** gave a measure of the relative numbers of the photo-electrons playing a part in the simultaneous emissions of the *K*, *L*, *M*., etc., types of spectra. That it is a complex phenomenon is apparent from the recent detailed investigation of H. Robinson.†† M. and L. de Broglie‡‡ have deduced that 84 per cent. of the energy absorbed is associated with the production of *K* electrons and the remainder with *L*, *M*, *N*., etc., electrons. They also direct attention to the question of whether the stream of *K* electrons itself possesses a greater or smaller amount of energy than that of the *L* electrons expelled by the *same* radiation. The view now generally adopted is that the energy of each *K* quantum and of each *K* electron associated with its emission are, as it were, complementary, the sum of the two energies being equal to that of the incident quantum of energy according to equation (1).

As if in direct opposition to these experimental facts, we have the following: The "*K*" wave spectrum predominates, and as the energy of the individual *K* electron is smallest we might expect on the whole a diminution in the aggregate energy of the corpuscles after the *K* absorption limit has been exceeded in frequency. We might expect, for example, a diminution in the ionization of a gas which is due entirely to corpuscular bombardment,§§ under these circumstances, yet the ionization relatively to air *increases* in the ratio 3·5 : 1, for example when the Br atom of C_2H_5Br is made to fluoresce with its *K* spectrum and the increase is independent (for a given number of Br atoms) of the nature of the atom absorbing the corpuscular energy.‖‖ At the same time there is an increase in the absorption coefficient of the same order. Precisely the same thing is true for the emission of photo-electrons from metals. There is a selective increase in the *total* energy of the corpuscles emitted, accompanied by a selective increase in the absorption coefficient for the incident X-rays. The two alternatives, therefore, are (*a*) when the *K* spectrum is excited the *same* number of atoms as before (when only the *L*, *M*, *N* spectra are excited) is playing a part in the corpuscular emission, and therefore the concomitant of the *K* spectral line must be a rapid corpuscle endowed with a *K* quantum of energy,

* Coster, Phil. Mag., Vol. 43, p. 1070 (1922).
† Phil. Mag., Vol. 19, p. 337 (1910).
‡ Phil. Mag., Vol. 20, p. 320 (1910).
§ Phil. Mag., Vol. 30, p. 745 (1915).
‖ Phil. Mag., Vol. 41, p. 120 (1921).
¶ Journal de Physique, VI (ii), p. 265 (1921). C. R., 172, p. 274 (1921).
** Phil. Mag., Vol. 43, p. 1116 (1922).
†† Proc. Roy. Soc., A, 104, p. 455 (1923).
‡‡ C. R., 173, p. 527 (1921).
§§ C. T. R. Wilson, loc. cit. Simons, Proc. Univ. Durh. Phil. Soc., 5 (i) p. 11 (1913). Jauncey, Phil. Mag., Vol. 48, p. 81 (1924).
‖‖ Simons, Proc. Univ. Durh. Phil. Soc., 5 (i), p. 11 (1913).

or (*b*) the selective increase in corpuscular energy under these circumstances is merely a statistical result arising from the now newly existing possibility (i.e., after the K absorption limit has been exceeded in frequency) of the breaking down taking place from the K level, together with an increased mathematical probability that the break down will occur at this level. This dual process would involve an additional number of orbits for the K emission. It could give rise to the observed increase in ionization and absorption, but there would remain over for explanation the mathematical probability alluded to.

The difficulty of reconciling the absorption and the ionization phenomena in the region of the absorption band with photo-electric emission phenomena, which undoubtedly are represented by equation (1) is very real. It has been dealt with recently in some detail by Barkla and Miss Dallas,* who state that the K ionization is not produced simply, or even principally, by electrons with energy $h(\nu-\nu_K)$. The K ionization may be produced by electrons with energy $h\nu$. While there is a possibility that many K electrons are emitted with energy $h(\nu-\nu_K)$, no suggested theory which takes them into consideration seems capable of explaining the K ionization quantitatively.

The new experiments described below, taken in conjunction with some results previously obtained, and also with those of Barkla and Miss Dallas, throw some light on this complex subject. Whilst the main question remains unanswered experimentally—viz., how are selective ionization and absorption accounted for quantitatively—yet it will be shown that the average energy per photo-electron does not make a selective jump when the K absorption limit is passed, and that if the probability question could be settled there would be no need to depart from the well-established Planck-Einstein photo-electric equation.

Experimental Methods.

Shearer† and the writer‡ have shown that when a mixed beam of X-rays is incident upon a metal, two types of electronic discharge occur. One type—viz., the true photo-electrons or β-rays—have already been mentioned; the other type consists of a copious stream of slowly moving particles or δ-rays, whose most frequent resultant energy corresponds to a fall of potential of only a few volts. Later the writer§ was successful in demonstrating the same effect when the incident X-rays were rendered more or less homogeneous by reflection from a secondary radiator.

The same procedure was adopted in the work described below. For this purpose two extremely thin films were employed, one of gold and the other of silver, laid down electrolytically on paraffin wax.

Table I shows the K and L absorption limits for silver and gold respectively. These two metals were subjected to X-rays over a range of wave-lengths varying from 388×10^{-11} cm. (K_α rays from Ba) to 871×10^{-11} cm. (K_α rays from Sr). This range of wave-lengths lies well within the region between the K and L absorption limits for gold, but overlaps the K absorption limit for silver. By studying the electronic discharges from silver produced by X-rays whose wave-lengths were

* Phil. Mag., Vol. 47, p. 1 (1924).
† Phil. Mag., Vol. 44, p. 793 (1922).
‡ Phil. Mag., Vol. 46, p. 473 (1923).
§ Phil. Mag., Vol. 48, p. 250 (1924).

round about that of the K absorption limit for silver, and comparing these discharges with those from gold under similar conditions as a control, it was hoped to determine something of the nature of the so-called "K electronic emission." All that can be stated about the thinness of the films is that the graphite upon which they were deposited could be seen through them. The employment of such extremely thin films eliminates as far as possible the necessity for taking into account many factors involved in the absorption of slow electrons in metals about which very little is known at present.*

TABLE I.

	Absorption limits, cm.	
	K	L
Silver	485×10^{-11}	c. 3500×10^{-11}
Gold	153×10^{-11}	c. 930×10^{-11}

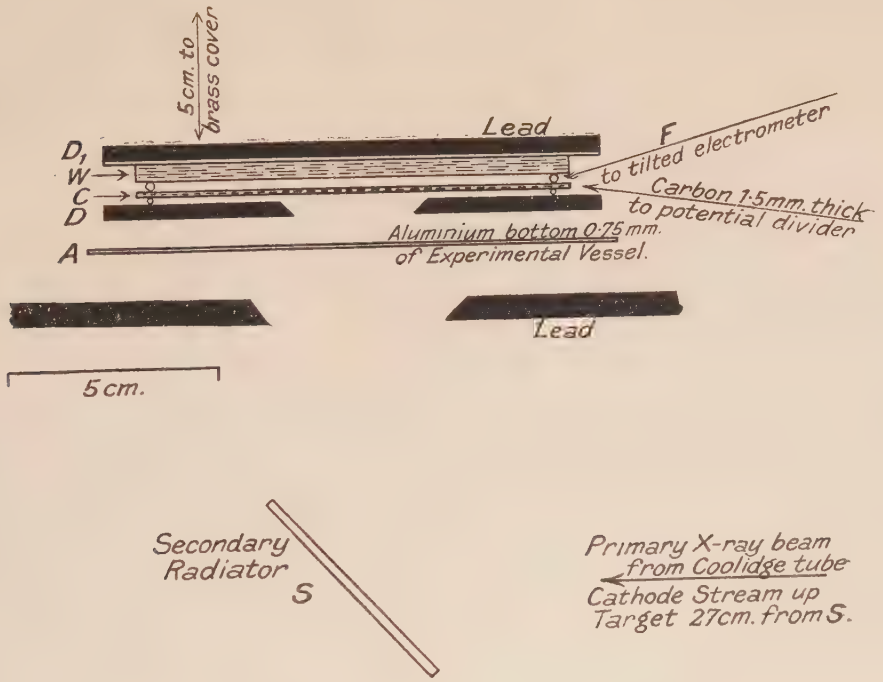


FIG. 1.

The arrangement of the silver or gold film with respect to the incident beam of X-rays is shown in Fig. 1. The primary beam of X-rays was produced by a tungsten target radiator pattern Coolidge tube worked by means of a transformer without rectifying devices. The current through the tube was three milliampères, and the peak potential measured between balls 10 cm. in diameter was 77,400 volts,

* Compton and Ross, Phys. Rev., Vol. 13, p. 374 (1919).

corresponding to a quantum limit of wave-length of 155×10^{-11} cm., or to a maximum effective wave-length, calculated from Ulrey's constant,* of 358×10^{-11} cm. The characteristic K radiation should, therefore, have been excited in all the secondary radiators employed at S. This radiation, together with any L, M, etc., radiation and scattered radiation, passed vertically upwards into the experimental vessel, of which A represents the outer aluminium bottom 0·75 mm. thick, then through a hole 3 cm. in diameter in the lead diaphragm D, through a thin carbon plate C 1·5 mm. thick, and finally fell on to the film F of silver or gold on the surface of the paraffin wax plate W, 4 mm. thick. The radiation was finally completely absorbed in the brass sheet and lead plate D_1, on top of the wax. The film F was connected to a Wilson's tilted electroscope, and the carbon plate C to a potential divider, the insulation between them being three small sulphur plugs 2 mm. high. The experimental chamber was exhausted by means of a mercury vapour pump to a mean pressure of 0·035 mm. of mercury, as measured by a Pirani pressure gauge. The factors deciding this pressure were the ease with which it could be established and maintained and the independence of electroscope readings on small fluctuations about this pressure. The electroscope recorded a positive charging of F, due to the emission of electrons from the film. The null method of using the instrument when the difference of potential between F and C was of the order 1 volt or less has been described elsewhere.† Its sensitiveness in order to obtain readable deflections had to be of the order 0·005 volt per division of the eyepiece scale, with a capacity of the system of about 40 cm. No changes whatever were made in the reading arrangements when the gold film was replaced by the silver film at F, and, as the X-ray generating devices worked very steadily, a time factor gave the quantity of X-rays falling upon the films. The time of run ranged between two and four minutes. It is presumed throughout that the softer constituents of the charateristic radiations from the secondary radiators have been absorbed by the aluminium and carbon, so that probably none reached the film.

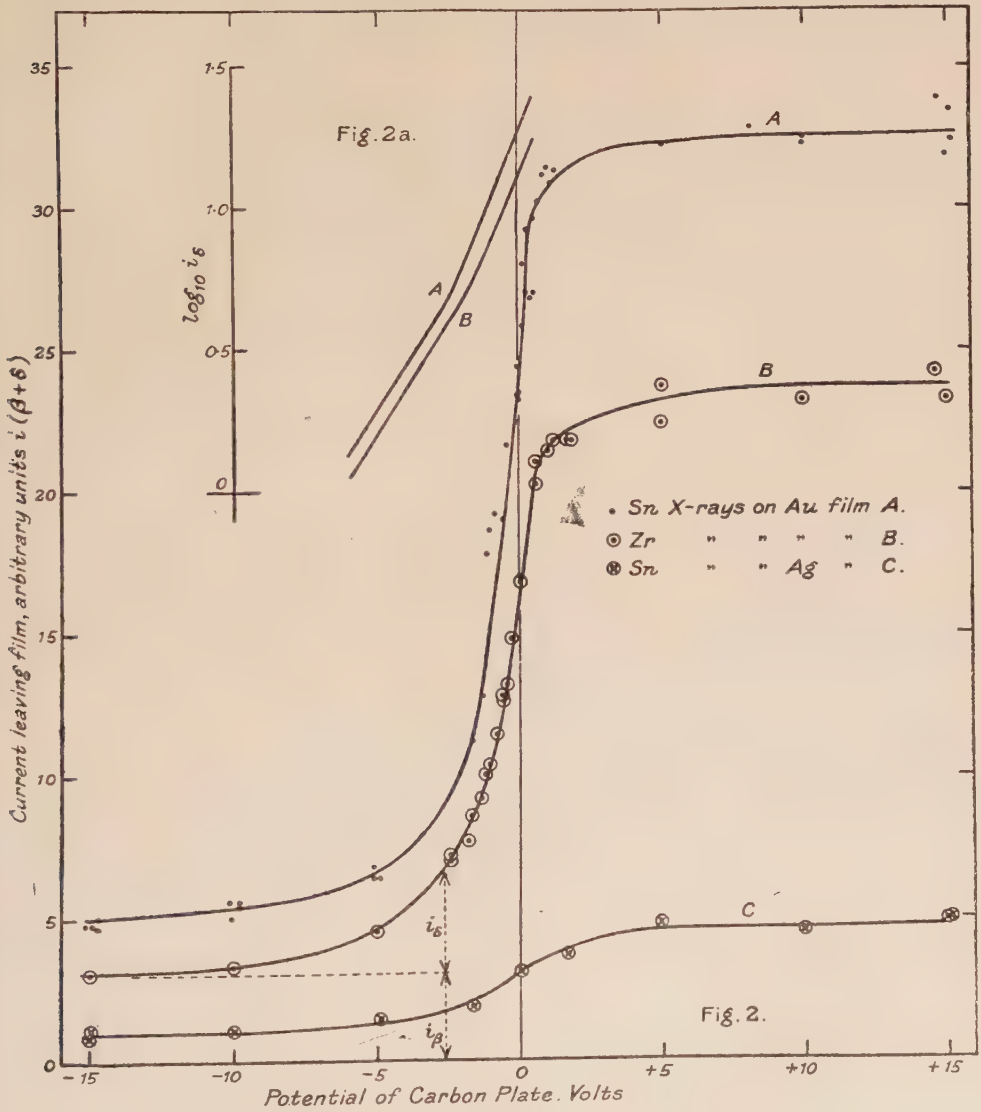
Examples of the type of curve obtained for the electronic current from the films plotted against the accelerating or retarding field are given in Fig. 2. The ordinates of these curves represent approximately the number of electrons emerging from the film with a normal energy component (at 180° to the direction of the incident X-ray beam) equal to or algebraically less than the values of the potential given by the abscissæ. The interpretation of that part of the curve on the right-hand side of the ordinate $V=0$ presents many difficulties. It is probable that quite a large proportion of the electrons remain entangled quite close to the surface, and can only be liberated by an accelerating field.

The Separation of the β and the δ-rays.

Throughout the range of incident wave-lengths employed it was observed that the curves for the electronic emissions from the films began to rise steeply in the region of about −10 volts, the wave-length of the incident rays and the nature of the screen having but little influence on the position of this change. This conforms with Shearer's observation‡ that "the nature of the screen has but little influence

* Phys. Rev., Vol. 11, p. 401 (1918).
† Simons, Phil. Mag., Vol. 46, p. 473 (1923).
‡ Phil. Mag., Vol. 44, p. 806 (1922).

upon the velocity distribution of the slow electrons from it." The velocity distribution of these slow electrons seems to be uninfluenced either by the nature of the metal from which they emerge or by the wave-length of the X-rays producing them. In Fig. 2 (*a*) the logarithms of the currents for curves *A* and *B* are plotted against

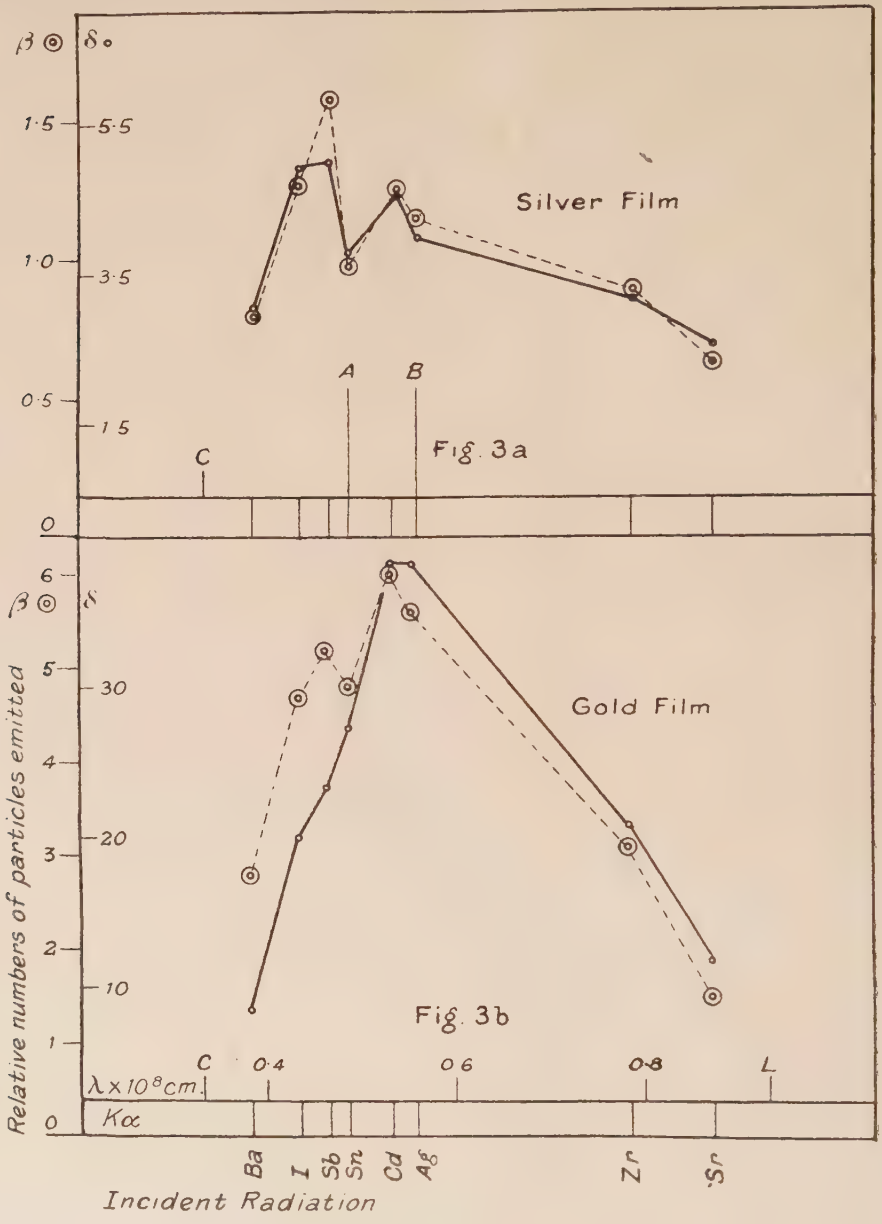


FIGS. 2 AND 2 (*a*).

potential. The most frequent resultant energy of these δ or slow rays, taken from the slope of these logarithmic curves, is of the order 3·5 volts.

There is a slow diminution in the number of electrons emerging from the screen as the retarding potential diminishes below −10 volts. This, together with the

general nature of the shape of the curves, mentioned in the preceding paragraph, leads to the conclusion that the region about —15 volts marks off the two distinct



FIGS. 3(*a*) AND 3(*b*).

groups of electrons. The ordinate in this region measures the total *number* of β particles escaping from the film, and the difference between the ordinates at —15

and 15 volts, say, measures in a similar manner the total *number* of δ particles which form a second distinct group. These are produced in an entirely different manner from the first.

The relative numbers of β and δ particles emerging from the silver and gold films for the incidence of X-rays of various wave-lengths and under precisely similar conditions are given in Table II. These values are plotted in Fig. 3 (*a*) for the silver

TABLE II.

K radiations from	Relative number of Particles emitted when incident upon			
	Silver Screen.		Gold Screen.	
	β-parts.	δ-parts.	β-parts.	δ-parts.
Sr λK_α=871 X.U. ...	0·63	2·57	1·5	11·9
Zr 788 ...	0·90	3·20	3·0	21·0
Ag 562 ...	1·15	3·99	5·6	38·4
Cd 538 ...	1·26	4·57	6·0	38·5
Sn 487 ...	0·98	3·79	4·8	27·6
Sb 468 ...	1·58	5·00	5·2	23·3
I 437 ...	1·27	4·94	4·7	20·0
Ba 388 ...	0·80	3·05	2·8	8·5

film and Fig. 3 (*b*) for the gold film. It is fully realised that the values of the ordinates of these curves involve many unknown factors, such as, for example, the distribution of intensity amongst wave-lengths of the primary radiation from the Coolidge tube, the relative intensity of the secondary radiation from a thick radiator to that of the same constituent contained in the primary, the scattered radiation from the secondary radiator, the influence of density and thickness of the secondary radiators employed, and chiefly the absorption of this incident radiation in the aluminium and carbon of the chamber; but all these are eliminated by the comparison of similarly situated ordinates in Figs. 3 (*a*) and 3 (*b*). This has been done in Fig. 4. In Figs. 3 (*a*) and 4 *AB* represents the breadth of the *K* absorption band, extending from $\lambda=0{\cdot}567\times10^{-8}$ cm. (K_α wave-length for silver) to $\lambda=0{\cdot}485\times10^{-8}$ cm. (*K* absorption limit for silver). *L* represents the approximate position of the *L* absorption limit for gold and *C* the position of the maximum effective output of the Coolidge tube.

The comparative smoothness of the gold curves, taken under exactly the same conditions as the silver curves above, indicates that the complexity of the latter in the region of the absorption band is real, or in other words, in spite of the necessarily complex nature of each curve, due to causes already indicated, both the shape of each, and the magnitude of its ordinates, are primarily controlled by the metal of the screen—viz., gold or silver respectively.

In Fig. 4 the curve marked β/β shows the ratio of the number of β particles emerging from the silver film to the number emerging from the gold film under precisely the same conditions, whilst the curve marked δ/δ shows in a similar manner the ratio of the δ particles emerging from the two films.

Generally it would appear that the transformation of X-rays into β and δ-rays as measured by their number, not their energy, falls to a minimum within the region of the so-called absorption band, and that there appears to be a special increase in the number of each type of particle on the short wave-length side of the region

of the absorption band. The increase is much more marked in the case of the β-particles than in that of the δ-particles. Remembering that it is the numbers of the β and δ-particles that have been dealt with up to the present, it is, nevertheless, surprising that the effects are not more marked in the region of the absorption band than they experimentally appear to be; the selective ionization in a gas beyond the region of the absorption band, the corresponding selective absorption and the emission of line spectra are all such well-marked phenomena.

THE PROPERTIES OF THE β-RAYS IN THE REGION OF THE ABSORPTION BAND.

In the Paper by Barkla and Miss Dallas previously referred to the authors have drawn a curve for

$$\frac{\text{total ionization in } H_2 \text{ by the complete absorption of } \beta\text{-parts . from Ag leaf}}{\text{,, ,, ,, ,, ,, ,, Pt ,,}} = \frac{{}_{Ag}E_\lambda}{{}_{Pt}E_\lambda}$$

when the β-particles are produced by X-rays of various wave-lengths. The numerical values of these ratios are not given, but this omission does not invalidate the argument that follows. By a suitable adjustment of the ordinate it is at once apparent and somewhat surprising, when it is considered how diverse the two experiments are, that their curve, the broken line in Fig. 4 marked *B* & *D*, shows undoubted similarity to the curve marked β/β obtained in this work. The points on this curve are

$$\frac{\text{number of } \beta\text{-particles emerging from a thin silver film}}{\text{,, ,, ,, ,, gold ,,}} = \frac{{}_{Ag}n_\lambda}{{}_{Au}n_\lambda}$$

the β-particles being produced by X-rays over the same range of wave-lengths as those of Barkla and Miss Dallas. Platinum and gold are so near to one another in atomic number, and sufficiently remote from silver, that they may, for purposes of this work, be regarded as equivalent. If we take it for granted that the differences between the dotted curve and the whole line curve at the upper part of Fig. 4 are only due to the great experimental difficulties in measuring the very small electric currents in this work, then we have, on dividing the above ratios by each other,

$$\frac{{}_{Ag}E_\lambda}{{}_{Ag}n_\lambda} \Big/ \frac{{}_{Au}E_\lambda}{{}_{Au}n_\lambda} = \frac{Af(\lambda)}{Bf(\lambda)}, \text{ where } A \text{ and } B \text{ are constants} = \text{constant.}$$

Omitting now from our considerations the X-ray energy that would escape from the hydrogen in the experiments of Barkla and Miss Dallas during the ionization by impact by the β-particles amongst the atoms, and taking ${}_{Ag}E_\lambda/{}_{Ag}n_\lambda = {}_{Ag}\bar{E}_\lambda =$ the average energy of a β-particle emerging from Ag under the influence of X-rays of wave-length λ, thus it follows that

${}_{Ag}\bar{E}_\lambda/{}_{Au}\bar{E}_\lambda =$ constant over the region of the absorption band and through the absorption limit for Ag.

In confirmation of this the author found in some previous work*

$$({}_{Ag}E/{}_{Au}E)_{0\cdot562\text{Å. U.}} = 0\cdot43\,; \quad ({}_{Ag}E/{}_{Au}E)_{0\cdot388\text{Å. U.}} = 0\cdot48 \text{ (values relative)}$$

when the β-rays were absorbed in air. The wave-lengths quoted are on either side

* Phil. Mag., Vol. 41, p. 120 (1921).

of the K absorption limit for silver, which is at 0·485Å.U. The present work gives

$$({}_{Ag}n/{}_{Au}n)_{0\cdot562\text{Å.U.}}=0\cdot21\ ;\quad ({}_{Ag}n/{}_{Au}n)_{0\cdot388\text{Å.U.}}=0\cdot28 \text{ (values relative).}$$

Hence

$$({}_{Ag}\bar{E}/{}_{Au}\bar{E})_{0\cdot562\text{Å.U.}}=2\cdot0\ ;\quad ({}_{Ag}\bar{E}/{}_{Au}\bar{E})_{0\cdot388\text{Å.U.}}=1\cdot7$$

The K, L, M, etc., corpuscles being excited from the silver in the latter case and only L, M, etc., corpuscles in the former case.

The approximate equality of the numbers 2 and 1·7 and the close similarity between the curves in the upper portion of Fig. 4 yielding the results of the last two paragraphs point strongly to the important conclusion that there is no *selective*

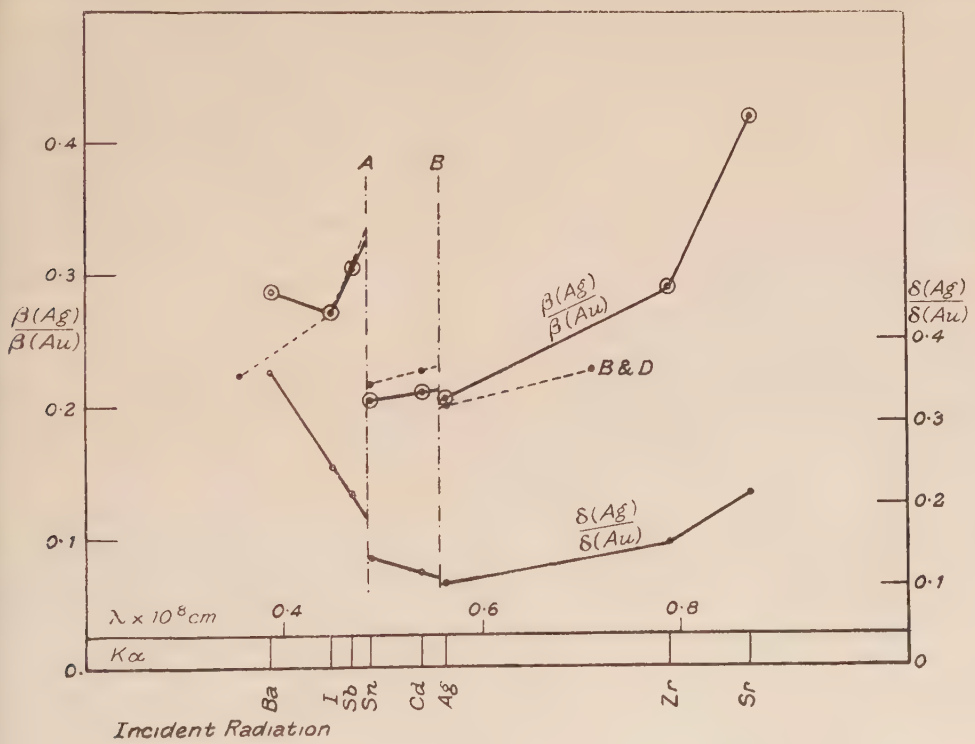


FIG. 4.

increase in the average energy of the β-rays emerging from the silver film as the region of the absorption limit of the X-rays incident upon it is passed across. It will be seen that if this conclusion can be generalised, selective increase in ionization and absorption does not involve a similar selective increase in the average individual energy of the electrons taking part in the action ; but it does involve a selective increase in the number of electrons participating. On the excitation of the K fluorescent spectrum there will be added to the resultant electron stream a comparatively large number of K electrons (of smaller individual energy than L, M, etc., electrons), which, ionizing by collision, produce the selective increase in ionization in the case of a gas. Selective absorption is the accompanying phenomenon

of the participation of a large number of additional atoms whose K levels are disturbed. Reference must here be made to two recent successful theoretical contributions, one by Kramers* and the other by Jauncey† on the subject of the change in the absorption coefficient over the region of the absorption limit. The treatment in both cases is statistical, and assumes that the K special absorption is allied to the increased probability of the interaction between the radiation and a new additional set of K orbits. This is the first direct confirmatory experimental evidence of the line of argument adopted by these two writers. The author attempted some years ago to obtain on statistical lines a theoretical expression for this change in the absorption coefficient.‡

The δ-rays.

Of the δ-rays produced throughout the thickness of the film, only those liberated quite close to the surface have a chance of escaping. This is shown by the great steepness of the curves of Fig. 2. Electrons which have a most frequent energy due to a fall through only a few volts cannot have been liberated in the first instance from deep molecular layers. We must therefore regard the δ-rays observed as a surface phenomenon. In his recent work, C. T. R. Wilson§ has calculated the value of the average energy of the δ-electrons liberated in a gas to be of the order 7 volts, whilst it is evident from the rounded shape of the curves of Fig. 2 on the right-hand side of the axis $V=0$ that the value of 3·5 volts for the most frequent energy of the δ-particles from a metal should have a comparable quantity added to it, due to the entrainment of a large number of the very slow particles in the surface of the film. It has already been remarked that neither the nature of the metal from which they come nor the wave-length of the X-rays producing them seems to have any influence upon the velocity distribution. The curves show that very few of the electrons of this group have a resultant energy greater than about 40 volts.

The presence of this group of particles is such an outstanding feature that I have been led to examine in some detail the method of their production. These particles, together with an extremely absorbable radiant energy, accompanying their production, must be regarded as the "end-products" of X-rays. As the velocity distribution curves show such a remarkable similarity to those for the thermions emitted from hot bodies as in Richardson's work, the experimental conditions being similar, the author carried out a numerical investigation,‖ assuming that the distribution in speeds was Maxwellian, and employed Richardson's formula for the normal components of the energy, viz.,

$$\log_e i/i_0 = -\frac{ne}{RT} \cdot V_1,$$

where V_1 is the retarding potential reducing the normal δ current from i_0 to i, ne $=0{\cdot}43$ e.m.u. and $R=3{\cdot}7\times10^3$ erg/deg.C.¶ In this manner a temperature T was

* Phil. Mag., Vol. 46, p. 836 (1923).
† Loc. cit.
‡ Trans. Roy. Soc. S. Africa, 6, 4, p. 311 (1917).
§ Loc. cit.
‖ Phil. Mag., Vol. 46, p. 473 (1923).
¶ "The Emission of Electricity from Hot Bodies," p. 144 (1916).

calculated which, on the hypothesis of the similarity between the two classes of phenomena, could be regarded as that due to atomic recoil of the centres yielding the primary β-particle, these hot centres momentarily becoming thermionic (δ-ray producing). But the temperature calculated from the slope of the log. lines of Fig. 2 (*a*), using the above formula, is altogether too high to be accounted for by atomic recoil consequent upon the ejection of even the most rapid β-particles available. The thermionic process does not offer any clue to the production of these extremely slow electrons.

The possibility suggests itself that the δ-electrons are themselves the recoil electrons, consequent upon the general scattering process of the incident X-rays. Though this is untenable for reasons stated below, yet the experimental results strongly suggest some relationship to the recently published quantum theory of scattering by Compton.* As a consequence of this theory, Compton and Hubbard† have deduced that the energy of the recoiling electron will be

$$K=\frac{2a h\nu_0 \cos^2\theta}{(1+a)^2-a^2\cos^2\theta}$$

in which $h\nu_0$ is the value of the incident quantum, $a=h\nu_0/mc^2$, m being the mass of an electron and c the velocity of light, θ is the angle between the direction of recoil of the electron and that of the incident quantum, the permissible values of θ ranging between 0° and 90° round the direction of the incident quantum. This expression varies as $\cos^2\theta$ for the small values of a satisfied in the present experiments. Now a thin film exposed normally to the incident beam of X-rays would emit from the faces of incidence and emergence a similar distribution of characteristic X-rays (i.e., characteristic of the film), each quantum of which, before passing away from either face, should stand an equal chance of being scattered. The recoil electrons due to the scattering of the characteristic rays should therefore be equally distributed on both sides of the film. This is not the case, however, when the incident beam is directly scattered. Those quanta which return at an angle of 180° to the direction of the incident quanta, thereby suffering a maximum degradation of energy, will yield a recoil electron of maximum energy in a direction making an angle $\theta=0°$ to that of the incident quantum, whilst those passing on unscattered will yield a recoil electron of zero energy in a direction of $\theta=90°$ round that of the incident quantum.

In the former of these two cases we have

$$K_{\max.}=h\nu_0 \cdot 2a/(1+2a),$$

and if we consider that the only recoil electrons we are concerned with in these experiments are those produced by the scattering of the X-rays characteristic of the gold film itself, we have ${}_{Au}\lambda_L=1{\cdot}273\times10^{-8}$ cm. $\therefore$ $a=0{\cdot}019$, making $K_{\max.}=357$ volts. It is scarcely probable, therefore, as the recoil electrons vary in energy from 357 volts down to zero, that practically all of them should lie within the range of energy 0 to 30 volts, with a most frequent energy of 3·5 volts, determined from the curves of Fig. 2 (*a*), and that the velocity distribution should be experimentally independent of the nature of the screen from which they emerge, which also enters

* Phys. Rev., Vol. 21, p. 486 (1923).
† Phys. Rev., Vol. 23, 4, p. 439 (1924).

into the above calculation. The same thing is true if we adopt the view expressed recently by C. T. R. Wilson,* in which the incident quantum is scattered uniformly in all directions. The maximum value of the recoil velocity is now u in the expression $h\nu_0 = cmu$, where $h\nu_0$ is the energy of the incident quantum. On this theory, too, the energy of the recoiling electrons would vary from about 96 volts downwards, and be dependent upon the metal of the screen. The experimental evidence is thus against the view that the δ-rays are recoil electrons.

Returning, therefore, to the view that the δ-rays are produced by the impact of the rapid β-rays on the atoms, it remains unexplained why the group should be so sharply marked, or, in other words, why only those electrons most loosely bound to the atoms should be removed by the impact of the energetic β-particle. This is one of the most remarkable features of the cloud tracks of both α and β-particles. Fig. 3 (*a*) shows clearly that there is a simple proportionality between the number of β-particles and the number of δ-particles produced by them as they emerge from a silver film. In the case of the gold film, Fig. 3 (*b*), this proportionality is not so marked. The difference between the two cases receives a tentative explanation from the following experimental observations. Dauvillier† states that the Compton effect—i.e., the increasing of the wave-length of X-rays after scattering—only appears in the case where the selective absorption is small, and C. T. R. Wilson‡ has formulated an hypothesis that characteristic X-rays can be transmitted by a "handing-on" process along the path of a β-particle. As the energy of the β-particles is so great, they would be imperceptibly influenced by the scattered X-rays at the moment of their emergence from the film. On the other hand, the impact electrons, on account of their small energy, may be profoundly influenced at the moment of emergence by the radiations scattered backwards from the face of incidence. The distribution of intensity amongst wave-lengths of this scattered radiation would be the same as that of the incident radiation, but it would suffer a general increase in wave-length given by§

$$\triangle\lambda = (2h/mc) \times (\text{mean value of } \sin^2 \varphi/2)$$

φ is the angle between the incident and emergent quanta. In these experiments the scattered rays emerge from the face of incidence of the X-rays on the film so that φ ranges over from 180° to 90°,

$$\therefore \triangle\lambda = 0{\cdot}04 \text{ Å.U.}$$

Figs. 3 (*a*) and 3 (*b*) exhibit all these effects. There is a lateral shift between the δ-ray and β-ray distribution curves from gold of approximately 0·04 Å.U. This effect is not shown in the case of the emissions from silver, because of the absorption band coming in the middle of the curve.

Birkbeck College,
University of London.

DISCUSSION.

Dr. D. Owen expressed admiration of the skilful disposition of apparatus used by the author. Having had the opportunity of actually seeing the apparatus as set up he felt certain that the measurements recorded in the Paper were amply worthy of careful consideration.

* Loc. cit.
† Nature, Vol. 114, p. 111 (1924).
‡ Loc. cit., p. 209.
§ Compton, Phys. Rev., Vol. 21, p. 486 (1923).

Owing to the lateness of the hour, the PRESIDENT closed the discussion with the remark that the Paper constituted a contribution of the highest value to the Proceedings of the Society. He asked that remarks by other intending speakers should be forwarded to the Secretary.

Prof. A. O. RANKINE (communicated) : As there is no sharp line of demarcation between the β and δ-rays would it not have been better to have taken as the measure of the β-ray emission the ordinate of Fig. 2 corresponding to the minimum ionizing potential of an atom instead of fixing upon the ordinate at the arbitrary point —15 volts ? Again in deducing the main result of the Paper use is made of two quantities, (*a*) the total energy of photo-electric emission as measured by the number of pairs of ions produced in a gas by the complete absorption of photo-electron energy, and (*b*) the number of photo-electrons engaged in ionizing. The former of these two quantities contains all the effects right down to any ionization produced by the slowest particles, whilst the latter excludes the possible ionizing effects of particles whose resultant velocity is less than 45 volts. Is it legitimate, therefore, to draw any conclusions from the ratio of (*a*) to (*b*) ?

AUTHOR'S reply (communicated) : These very apposite questions have received careful consideration during the writing of the Paper. The ordinate at —15 volts in Fig. 2 measures the total number of β-rays present whose *resultant* velocity is greater than 45 volts. The choice of this point has been considerably influenced by Shearer's curve (Phil. Mag. 44, p. 806 (1922)), which has been carried down to —180 volts, by C. T. R. Wilson's remarks (Proc. Roy. Soc., A. 104, p. 211(1923)), and by the constant similarity between all the curves I have taken. The ordinate I have taken, viz., that at the foot of the *only* steep fall that exists, so far as I know, along the current-potential curve marks the dividing line between the number of primary β-particles and of the secondary effects produced by them. These secondary effects include the production of electrons of all speeds up to 45 volts, which themselves may produce on the average 3 or 4 more electrons each which cannot ionize further. An essential feature of the work has been to sort out the true, primary, high-energy, β-ray emission from all the low-energy secondary collision effects produced by them.

C. T. R. Wilson has shown that the primary ionization is about 90 pairs of ions per cm. in air for β-rays of velocity about 10^{10} cm. per sec., which is a rough estimate of the velocity of the primary β-rays whose numbers have been measured in this Paper. The total ionization per cm. including that due to the secondary β-particles is about 3 or 4 times as large as the primary.

It is in view of these considerations that I have chosen the ordinate I have as measuring the number of primary β-rays, whilst the other references in the Paper to the work of Barkla and Miss Dallas and to my own previous results undoubtedly give the *total* energy of these particles. Perhaps it would have been better had I said that there was no evidence of a selective increase in the average energy of the *primary* β-particle over the region of the absorption limits.

DEMONSTRATION OF AN ELECTRICAL METHOD OF PRODUCING VOWEL SOUNDS AND ITS APPLICATION TO WIRELESS TELEGRAPHY.

By Dr. W. H. Eccles, *F.R.S.*, and C. F. A. Wagstaffe.

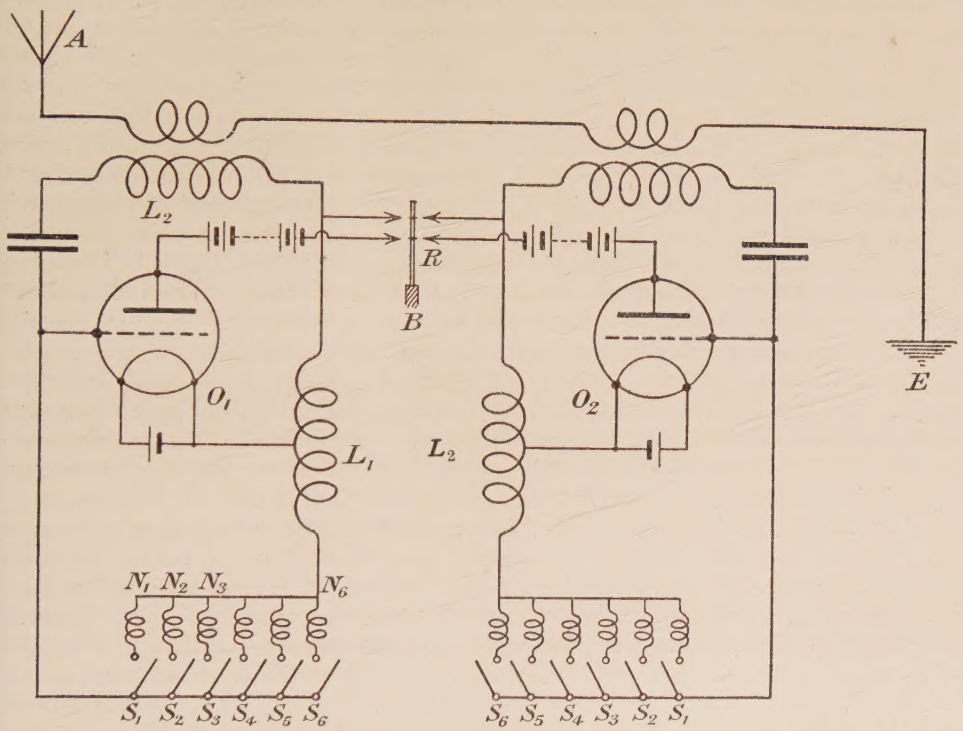
THE apparatus to be shown represents wireless sending and receiving stations. It was devised to test a proposed method of communication in which the electric waves transmitted could produce easily recognizable chords at a suitably equipped receiving station. It was thought that chords would be more easily recognised than simple tones, and that vowel sounds would be the most easily recognised of chords.

The ideal aimed at in this method of communication is that an operator at one side of the ocean should converse with an operator at the other side by means of a language made up of vowel sounds (somewhat like the Hawaian language), the sounds being produced by pressing keys. Any ordinary language could be converted immediately into this telegraphic form by a code wherein each consonant was represented by a definite pair of vowel sounds.

Consider the problem of transmitting the vowel *ah* from a wireless station utilising a frequency of, say, 100,000 cycles per second. One way is to pronounce the vowel in front of a microphone arranged for wireless telephony. But if the human voice is not to be employed, if, in fact, an artificial vowel sound is to be produced by pressing a key, vibrations of a voice frequency such as 250 cycles per second must be combined, according to the analysis of Sir Richard Paget, with the characteristic frequencies 800 and 1,300. Then these must be impressed upon the oscillations of 100,000 frequency by a microphone or otherwise. The simplest electrical solution is to employ a buzzer of frequency 250 cycles per second to excite at each stroke two circuits tuned respectively to 800 and 1,300 cycles per second, and to pass the complex currents through a telephone to convert them to sound waves. Such an apparatus has been used for testing telephones by the Western Electric Co. without any thought of applying it to wireless signalling. For this latter purpose the sound from the telephone could be applied to a microphone, or, alternatively, the complex currents could be used directly to modulate the high-frequency oscillations.

The method to be described is, however, more direct than that just suggested, and currents of acoustic frequency need not appear at any stage earlier than in the telephones of the receiving operator. Broadly, the method is as follows: At the transmitting station the pressing of a key produces immediately electrical oscillations of frequencies 100,800 and 101,300, and waves of these two frequencies are radiated. At the receiving station the waves produce corresponding oscillations, and these are combined with a locally generated frequency of 100,000 cycles per second in the well-known "heterodyne" manner so as to produce currents of frequencies 800 and 1,300. In order that these should together simulate the vowel sound *ah*, a current of voice frequency, say, 250 cycles per second, should be superimposed in a suitable way; this may be done either before the oscillations leave the sending station or after they reach the receiving station. Each method has given good results with a variety of vowels, made up by taking Paget's table of vowel constituents.

Three distinct ways of carrying out the above conception have been tried successfully. In the first of these ways a pair of oscillating circuits, which may be coupled together in any manner, or which may be completely separated, are adjusted to have the two natural frequencies of, say, 100,800 and 101,300, and are excited by "musical spark plant of, say, 250 cycles per second. The consequent oscillations are passed to the sending antenna. If the two circuits are coupled and excited by one and the same spark, the two frequencies natural to the coupled circuits are partly determined by the coupling. At the receiving stations the oscillations are amplified if necessary and heterodyned by aid of a local oscillator of



A Antenna. *E* Earth. O_1 Triode Oscillator for Upper Frequencies. O_2 Triode Oscillator for Lower Frequencies.

frequency 100,000 per second. The resulting compound current is rectified in any manner and passed to the operator's telephones. The tone heard there possesses the quality of the vowel *ah*. Other vowels can be made similarly by associating other high-frequency circuits with the same spark apparatus, and thus intelligible signals can be transmitted by aid of a code. Instead of the heterodyne the apparatus known as the tone wheel and other apparatus may possibly be employed.

The second way of working employs two sources of continuous waves, one of frequency 100,800, say, and the other of frequency 101,300 to produce waves. The sources are modulated or interrupted at equal acoustic frequencies of, say, 250 per second, either by the same or different pieces of apparatus. The waves of frequency 100,800 may be emitted at the same instant of time as those of frequency 101,300, or they may be emitted alternately, as may be advantageous in cases where one

and the same interrupting apparatus is applied to both sources of continuous waves. In this latter event a physiological phenomenon which may be called the persistence of audition has been discovered and is relied upon to blend the results at the receiving station. Reception is carried out by heterodyning at 100,000 per second or by an analogous process. The sound heard in the telephones has the quality of the vowel *ah* superposed upon a "voice" of frequency 250 per second. The figure shows diagrammatically apparatus suitable for this latter way of working when six vowel sounds are to be produced and employed. It consists of two similar triode oscillators O_1, O_2, each capable of oscillating at any one of six chosen frequencies. The choice of frequency is given by the switches S_1, . . S_6. When S_1 is closed, the inductance in the left-hand oscillating circuit is made up of the coils L_1 and L_2 and N_1; when S_2 is closed, N_2 is substituted for N_1; and when S_3 is closed, N_3 is substituted. The respective circuits so formed are conveniently tuned to the desired frequencies by moving the N coils with respect to the L coils whereby the mutual inductance between the respective pairs is varied. Alternatively, small condensers have been used instead of the N coils. A buzzer B carrying a contact rod R is maintained in vibration at 250 per second and alternately stops and starts the oscillators by making and breaking their circuits. The two switches marked S_1 are closed simultaneously, preferably through insulating linkages, when a key (not shown in the figure) is depressed; the switches S_2 are similarly closed by another key, and switches S_3 by a third key. Thus by pressing one of the three keys the operator can transmit in one operation the two components which make up a "vowel" signal. It is found practicable to build and work apparatus of this type employing six vowel sounds familiar in the English language. By manipulation of the keys consonantal effects can be produced. It is not necessary to adhere to the vowel sounds of one language, or, indeed, to the vowel sounds of any existing language, in order to make easily recognisable complex sounds.

The third example of this class of apparatus employs two sources of continuous waves, one of frequency 100,800, say, and the other of frequency 101,300, to produce waves, and these are transmitted from the sending station without modulation. At the receiving station the received oscillations are heterodyned by a local oscillator of frequency 100,000 whose oscillations are modulated by local apparatus at a frequency of, say, 250 per second, and the result rectified and passed to the telephones. The "voice" heard in the telephones is of frequency 250 per second, and the vowel quality, being determined by the heterodyned high-frequency oscillations, is that of the vowel *ah*.

In the apparatus exhibited there is provided an oscillatory circuit of acoustic frequency which has been calibrated so that various settings of the condenser give to the circuit the twelve acoustic frequencies required for the six vowels. This circuit is employed as an electrical resonator for the adjustment of the heterodyne currents accurately to their correct frequencies.

DISCUSSION.

Sir RICHARD PAGET said that the Demonstration proved conclusively that effects of the kind described can actually be obtained. He would suggest experimenting with the simplest form of electrical sound-producers on the lines followed by Mr. J. Q. Stewart, of the Western Electric Company of America, in the testing of telephone receivers. In such apparatus low-frequency resonant circuits are employed and the necessity for heterodyning is avoided; and if, as the speaker believed, consonant sounds consist of not more than four components, it should not be difficult to synthesise ordinary speech in a telephone. Such apparatus would not, of course, be applicable directly to telegraphy, but would be of considerable theoretical interest.

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